

Strong decays of the XYZ states

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Through the spin rearrangement scheme in the heavy quark limit, we have performed a comprehensive investigation of the decay pattern and production mechanism of the hidden beauty di-meson states, which are either composed of a P-wave bottom meson and an S-wave bottom meson or two S-wave bottom mesons. We further extend the corresponding formula to discuss the decay behavior of some charmonium-like states by combining the experimental information with our numerical results. The typical ratios presented in this work can be measured by future experiments like BESIII, Belle, LHCb and the forthcoming BelleII, which shall provide important clues to the inner structures of the exotic states.

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I. INTRODUCTION

Many so called XYZ states have been discovered in the past decade by the Belle, BaBar, CLEO-c, CDF, D0, CMS, LHCb and BESIII collaborations [1–3]. Among these observed XYZ states, some of them are good candidates of the exotic states. Especially, the charged charmonium-like state $Z_1(4475)$ was firstly observed by Belle [4, 5] and recently confirmed as a genuine resonance by LHCb [6]. Due to the peculiarity of $Z_1(4475)$, the number of the quark component of $Z_1(4475)$ is at least four. Up to now, $Z_1(4475)$ seems to be the best candidate of the four-quark states.

These XYZ states have stimulated theorist's extensive interest in revealing their underlying structures. Various theoretical schemes were proposed, which include the "exotic" explanations like the tetraquark state, molecular state and charmonium hybrid and non-resonant interpretations like the cusp effect and initial single pion emission mechanism [7–20].

Let's take the two charmonium-like states $X(3872)$ and $Y(4260)$ as an example. Since the mass of $X(3872)$ [21] is slightly below the $D\bar{D}^*$ threshold, the $D\bar{D}^*$ molecular picture was proposed in Refs. [22–31]. The charmonium-like state $Y(4260)$ was reported by BaBar in the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ process [33]. Later many theoretical explanations were proposed, which include the traditional charmonium assignment [34–37], charmonium hybrid [16–18], diquark-antidiquark state [38, 39], $D_1\bar{D}^*$ molecule or other molecular state assignments [40–45], and non-resonant explanation [46].

In order to answer whether these XYZ states can be explained as the candidate of the exotic states, one need carry out the dynamical calculation by adopting the specific dynamical model. For example, the one boson exchange model is often

applied to study the loosely molecular state.

On the other hand, the symmetry analysis, which does not depend on the dynamical model, can be an effective approach to explore the molecular state. The spin rearrangement scheme based on the heavy quark symmetry provides another approach to shed light on the inner structures of the XYZ states through investigating their decay and production behaviors. There were some discussions on $Z_c(3900)$, $Z_c(4025)$ and $Z_b(10610)/Z_b(10650)$ using the spin rearrangement scheme in the heavy quark limit [47–49]. The selection rules in the meson-antimeson states under the heavy quark symmetry were discussed in Ref. [50]. The relations between the rates of the radiative transitions from $\Upsilon(5S)$ to the hypothetical isovector molecular bottomonium resonances with negative G -parity via the spin rearrangement scheme were presented in Ref. [51].

Very recently, Ma *et al.* discussed the radiative decays of the XYZ states [52, 53], where the spin rearrangement scheme in the heavy quark limit was adopted. Besides their radiative decays, the strong decay patterns and production behaviors of the XYZ states are crucial to probe their inner structures. The experimental information of the strong decay modes of XYZ states is more abundant than that of the radiative decays of the XYZ states.

In this work we will adopt the spin rearrangement scheme and extend the formalism in Refs. [52, 53] to study the strong decays of the XYZ states. We will consider the following three classes of strong decays

$$\begin{aligned} B_{(1,2)}\bar{B}^{(*)} &\rightarrow (b\bar{b}) + \text{light meson}, \\ (b\bar{b}) &\rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}, \\ B_{(1,2)}\bar{B}^{(*)} &\rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}. \end{aligned}$$

corresponding to the strong decays from one molecular (resonant) state into a bottomonia, a bottomonia decaying into a molecular/resonant state, and strong decays from one molecular/resonant state into another molecular/resonant state, respectively, where we use the notations $B_{(1,2)}\bar{B}^{(*)}$ and $(b\bar{b})$ to denote the molecular/resonant states and bottomonium, respectively.

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This paper is organized as follows. After the introduction, we give the calculation details of the above three classes of strong decays in Sec. II. We present the numerical results in Sec. III. With the same method, we discuss the possible hidden-charm molecules/resonances in Sec. IV. The last section is the summary.

II. THE FORMALISM OF THE STRONG DECAYS IN THE HEAVY QUARK LIMIT

Heavy quark symmetry is a good tool in the study of the structures of hadrons containing heavy quarks. In the heavy quark limit, heavy quarks only have spin-independent chromoelectric interactions with gluons, while the spin-dependent chromomagnetic interaction is proportional to $1/m_Q$ and suppressed. Thus, the conserved angular momentum operators of a hadron containing the heavy quarks are the total angular momentum J , spin of heavy quarks S_H , which are also named as the "heavy spin" and the spin of light degrees of freedom S_l with the definition $\vec{S}_l \equiv \vec{J} - \vec{S}_H$. The spin of the light degrees of freedom includes all the orbital angular momenta and spin of light quarks within a hadron, where we simply denote it as the "light spin" in the following.

In this work, we investigate the strong decays of the hidden beauty molecular/resonant states composed of two bottom meson. We discuss two dimeson systems. The first one is a molecular/resonant state which is composed of one P-wave bottom meson like B_0, B'_1, B_1, B_2 and one S-wave bottom meson like \bar{B}, \bar{B}^* . The other is a molecular/resonant state composed of two S-wave bottom mesons.

In the heavy quark limit, $(B, B^*), (B_0, B'_1)$ and (B_1, B_2) belong to the doublets $H = (0^-, 1^-)$, $S = (0^+, 1^+)$ and $T = (1^+, 2^+)$, respectively. Adopting the same definition of the C -parity eigenstate of the molecular states in Refs. [50, 52, 53], we list all relevant molecular/resonant states in Table I. Since in this work we introduce no dynamical models, we will call them the "hidden beauty molecules/resonances" for simplicity. In the spin rearrangement scheme, the decay patterns of a hadron is determined by their spin configurations which are only determined by the spin structures of their constituents.

In the strong decays of hadrons containing the heavy quarks, not only the heavy spin, light spin, total angular momentum, C -parity and parity, but also G -parity and isospin are conserved. We need to distinguish the different isospin of a system. The $B_{(1,2)}\bar{B}^{(*)}$ system with one P-wave bottom meson and one S-wave bottom meson can be categorized as the isovector and isoscalar states with the corresponding spin

TABLE I: The hidden beauty molecular states with different J^{PC} quantum numbers.

J^{PC}	States		
1^{--}	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$
1^{-+}	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$
1^{++}	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$		
1^{+-}	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	$B^*\bar{B}^*$	
0^{--}	$\frac{1}{\sqrt{2}}(B_0\bar{B} - B\bar{B}_0)$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$
0^{-+}	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$
0^{++}	$B\bar{B}$	$B^*\bar{B}^*$	
2^{--}	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)$
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$		
2^{-+}	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)$
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$		
2^{++}	$B^*\bar{B}^*$		

wave functions

$$\begin{aligned}
|B_{(1,2)}\bar{B}^{(*)}\rangle_1^+ &= \frac{1}{\sqrt{2}}[|B_{(1,2)}^+\bar{B}^{(*)0}\rangle + \tilde{c}|B^{(*)+}\bar{B}_{(1,2)}^0\rangle], \\
|B_{(1,2)}\bar{B}^{(*)}\rangle_1^- &= \frac{1}{\sqrt{2}}[|B_{(1,2)}^0B^{(*)-}\rangle + \tilde{c}|B^{(*)0}\bar{B}_{(1,2)}^-\rangle], \\
|B_{(1,2)}\bar{B}^{(*)}\rangle_1^0 &= \frac{1}{2}\left\{[|B_{(1,2)}^0\bar{B}^{(*)0}\rangle - |B_{(1,2)}^+B^{(*)-}\rangle] \right. \\
&\quad \left. + \tilde{c}[|B^{(*)0}\bar{B}_{(1,2)}^0\rangle - |B^{(*)+}\bar{B}_{(1,2)}^-\rangle]\right\}, \\
|B_{(1,2)}\bar{B}^{(*)}\rangle_0^0 &= \frac{1}{2}\left\{[|B_{(1,2)}^0\bar{B}^{(*)0}\rangle + |B_{(1,2)}^+B^{(*)-}\rangle] \right. \\
&\quad \left. + \tilde{c}[|B^{(*)0}\bar{B}_{(1,2)}^0\rangle + |B^{(*)+}\bar{B}_{(1,2)}^-\rangle]\right\},
\end{aligned}$$

where $\tilde{c} = c(-1)^{L+K-J}$ and $c = \pm 1$ corresponds to C -parity $C = \mp$. The factor $(-1)^{L+K-J}$ is due to the exchange of the spin vectors of the two bottoms in the systems. The spin-flavor wave functions of the $B\bar{B}$ systems can be constructed as

$$\begin{aligned}
|B\bar{B}\rangle_1^+ &= |B^+\bar{B}^0\rangle, \\
|B\bar{B}\rangle_1^- &= |B^-\bar{B}^0\rangle, \\
|B\bar{B}\rangle_1^0 &= \frac{1}{\sqrt{2}}[|B^+\bar{B}^-\rangle - |B^0\bar{B}^0\rangle], \\
|B\bar{B}\rangle_0^0 &= \frac{1}{\sqrt{2}}[|B^+\bar{B}^-\rangle + |B^0\bar{B}^0\rangle].
\end{aligned}$$

The spin-flavor wave functions of the $B^*\bar{B}^*$ system can be cat-

egorized as

$$\begin{aligned}
|B^* \bar{B}^*\rangle_1^+ &= |B^{*+} \bar{B}^{*0}\rangle, \\
|B^* \bar{B}^*\rangle_1^- &= |B^{*-} \bar{B}^{*0}\rangle, \\
|B^* \bar{B}^*\rangle_1^0 &= \frac{1}{\sqrt{2}}[|B^{*+} \bar{B}^{*-}\rangle - |B^{*0} \bar{B}^{*0}\rangle], \\
|B^* \bar{B}^*\rangle_0^0 &= \frac{1}{\sqrt{2}}[|B^{*+} \bar{B}^{*-}\rangle + |B^{*0} \bar{B}^{*0}\rangle],
\end{aligned}$$

We decompose the total angular momentum of the above systems into their heavy spin and light spin. We adopt the spin re-coupling formula in analyzing the general spin structure as in Refs. [48, 52, 53]. For instance, we can decompose each part in the isoscalar states of the $B_{(1,2)} \bar{B}^{(*)}$ system, i.e.,

$$\begin{aligned}
|B_{(1,2)}^0 \bar{B}^{(*)0}\rangle &= \left[[\bar{b} \otimes (d \otimes 1)_s]_K \otimes [b \otimes \bar{d}]_L \right]_J |(\bar{b}d)(b\bar{d})\rangle \\
&= \sum_{g=0}^1 \sum_{m=0}^1 \sum_{h=|s-\frac{1}{2}|}^{s+\frac{1}{2}} \mathcal{A}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left(\left[d\bar{d} \right]_m \otimes 1 \right)_h \right]_J \right\rangle |(\bar{b}d)(b\bar{d})\rangle, \\
|B_{(1,2)}^{(*)0} \bar{B}^0\rangle &= \left[[\bar{b} \otimes q]_L \otimes [b \otimes (\bar{d} \otimes 1)_s]_K \right]_J |(\bar{b}d)(\bar{b}d)\rangle \\
&= \sum_{g=0}^1 \sum_{m=0}^1 \sum_{h=|s-\frac{1}{2}|}^{s+\frac{1}{2}} \mathcal{B}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left(\left[d\bar{d} \right]_m \otimes 1 \right)_h \right]_J \right\rangle |(\bar{b}d)(\bar{b}d)\rangle, \\
|B_{(1,2)}^+ B^{(*)-}\rangle &= \left[[\bar{b} \otimes (u \otimes 1)_s]_K \otimes [b \otimes \bar{u}]_L \right]_J |(\bar{b}u)(b\bar{u})\rangle \\
&= \sum_{g=0}^1 \sum_{m=0}^1 \sum_{h=|s-\frac{1}{2}|}^{s+\frac{1}{2}} \mathcal{A}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left([u\bar{u}]_m \otimes 1 \right)_h \right]_J \right\rangle |(\bar{b}u)(b\bar{u})\rangle, \\
|B_{(1,2)}^{(*)+} B_{(1,2)}^-\rangle &= \left[[\bar{b} \otimes u]_L \otimes [b \otimes (\bar{u} \otimes 1)_s]_K \right]_J |(\bar{b}u)(\bar{b}u)\rangle \\
&= \sum_{g=0}^1 \sum_{m=0}^1 \sum_{h=|s-\frac{1}{2}|}^{s+\frac{1}{2}} \mathcal{B}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left([u\bar{u}]_m \otimes 1 \right)_h \right]_J \right\rangle |(\bar{b}u)(\bar{b}u)\rangle,
\end{aligned}$$

In the above equations, the indices $b, \bar{b}, u, d, \bar{u}$ and \bar{d} in the square brackets represent the corresponding quark spin wave functions. The notation $[\bar{b} \otimes (q_i \otimes 1)_s]_K \otimes [b \otimes \bar{q}_j]_L \Big|_J$ denotes the spin structures of $B_{(1,2)} \bar{B}^{(*)}$. In the heavy quark limit, the spin of the light quark $q_i(u, d)$ in $B_{(1,2)}$ couples with the P-wave orbital angular momentum to form the light spin s , which further couples with \bar{b} to form the total angular momentum K . Similarly, the spin of the light quark $\bar{q}_j(u, d)$ in $\bar{B}^{(*)}$ couples with b , which corresponds to the total angular momentum L . Then, the coupling between K and L leads to the total angular momentum J of the systems. The notation $[[\bar{b}b]_g \otimes ([u\bar{u}]_m \otimes 1)_h]_J$ means that the two heavy quark spin \bar{b} and b couple into the heavy spin g and the two light quark spin u and \bar{u} couple into the total light quark spin m . And then, the coupling of m with the orbital angular momentum

from the P-wave bottom meson forms the light spin h . The spin re-coupling coefficients $\mathcal{A}_{g,m,h}^{s,L,K,J}$ and $\mathcal{B}_{g,m,h}^{s,L,K,J}$ are the same as that in Ref. [52].

We need to emphasize that we explicitly include the flavor wave function $|(\bar{b}q_i)(b\bar{q}_j)\rangle$ in Eq. (1). The symbol $|(\bar{b}q_i)\rangle$ represents $|(\bar{b}q_i) \equiv \frac{1}{\sqrt{2}}(|\bar{b}q_i\rangle + |q_i\bar{b}\rangle)\rangle$. Here, the position ordering of the \bar{b} and b cannot be interchanged in order to guarantee the orthogonalization of the heavy meson and anti-meson wave functions at the quark level. This treatment ensures the normalization of the spin configurations after performing the spin rearrangement, which was discussed in details in Ref. [52].

The spin structure of the isoscalar states of the $B_{(1,2)} \bar{B}^{(*)}$ systems can be expressed as

$$\begin{aligned}
|B_{(1,2)} \bar{B}^{(*)}\rangle_0^0 &= \frac{1}{\sqrt{2}} \sum_{g,m,h} \left\{ \mathcal{A}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left(\left[\frac{d\bar{d} + u\bar{u}}{\sqrt{2}} \right]_m \otimes 1 \right)_h \right]_J \right\rangle \right. \\
&\quad \times \left(\frac{|(\bar{b}d)(b\bar{d})\rangle + |(\bar{b}u)(b\bar{u})\rangle}{\sqrt{2}} \right) \\
&\quad \left. + \tilde{\mathcal{B}}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left(\left[\frac{d\bar{d} + u\bar{u}}{\sqrt{2}} \right]_m \otimes 1 \right)_h \right]_J \right\rangle \right. \\
&\quad \left. \times \left(\frac{|(b\bar{d})(\bar{b}d)\rangle + |(b\bar{u})(\bar{b}u)\rangle}{\sqrt{2}} \right) \right\}.
\end{aligned}$$

Similarly, we obtain the re-coupled spin structures of the isovector states of the $B_{(1,2)} \bar{B}^{(*)}$ systems as

$$\begin{aligned}
|B_{(1,2)} \bar{B}^{(*)}\rangle_1^0 &= \frac{1}{\sqrt{2}} \sum_{g,m,h} \left\{ \mathcal{A}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left(\left[\frac{d\bar{d} - u\bar{u}}{\sqrt{2}} \right]_m \otimes 1 \right)_h \right]_J \right\rangle \right. \\
&\quad \times \left(\frac{|(\bar{b}d)(b\bar{d})\rangle - |(\bar{b}u)(b\bar{u})\rangle}{\sqrt{2}} \right) \\
&\quad \left. + \tilde{\mathcal{B}}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left(\left[\frac{d\bar{d} - u\bar{u}}{\sqrt{2}} \right]_m \otimes 1 \right)_h \right]_J \right\rangle \right. \\
&\quad \left. \times \left(\frac{|(b\bar{d})(\bar{b}d)\rangle - |(b\bar{u})(\bar{b}u)\rangle}{\sqrt{2}} \right) \right\},
\end{aligned}$$

$$\begin{aligned}
|B_{(1,2)} \bar{B}^{(*)}\rangle_1^+ &= \frac{1}{\sqrt{2}} \sum_{g,m,h} \left\{ \mathcal{A}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left([u\bar{d}]_m \otimes 1 \right)_h \right]_J \right\rangle \right. \\
&\quad \left. + \tilde{\mathcal{B}}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left([u\bar{d}]_m \otimes 1 \right)_h \right]_J \right\rangle \right\} |(\bar{b}u)(b\bar{d})\rangle,
\end{aligned}$$

and

$$\begin{aligned}
|B_{(1,2)} \bar{B}^{(*)}\rangle_1^- &= \frac{1}{\sqrt{2}} \sum_{g,m,h} \left\{ \mathcal{A}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left([-d\bar{u}]_m \otimes 1 \right)_h \right]_J \right\rangle \right. \\
&\quad \left. + \tilde{\mathcal{B}}_{g,m,h}^{s,L,K,J} \left| \left[[\bar{b}b]_g \otimes \left([u\bar{d}]_m \otimes 1 \right)_h \right]_J \right\rangle \right\} |-(\bar{b}d)(b\bar{u})\rangle.
\end{aligned}$$

In heavy quark limit, the bottomonia can also be decomposed into the heavy spin and light spin

$$|\eta_b(1^1S_0)\rangle = |(0_H^- \otimes 0_l^+)_{0^{++}}\rangle|(b\bar{b})\rangle, \quad (1)$$

$$|\Upsilon(1^3S_1)\rangle = |(1_H^- \otimes 0_l^+)_{1^{--}}\rangle|(b\bar{b})\rangle, \quad (2)$$

$$|h_b(1^1P_1)\rangle = |(0_H^- \otimes 1_l^-)_{1^{+-}}\rangle|(b\bar{b})\rangle, \quad (3)$$

$$|\chi_{b0}(1^3P_0)\rangle = |(1_H^- \otimes 1_l^-)_{0^{++}}\rangle|(b\bar{b})\rangle, \quad (4)$$

$$|\chi_{b1}(1^3P_1)\rangle = |(1_H^- \otimes 1_l^-)_{1^{+-}}\rangle|(b\bar{b})\rangle, \quad (5)$$

$$|\chi_{b2}(1^3P_2)\rangle = |(1_H^- \otimes 1_l^-)_{2^{++}}\rangle|(b\bar{b})\rangle, \quad (6)$$

$$|\eta_{b2}(1^1D_2)\rangle = |(0_H^- \otimes 2_l^+)_{2^{+-}}\rangle|(b\bar{b})\rangle, \quad (7)$$

$$|\Upsilon(1^3D_1)\rangle = |(1_H^- \otimes 2_l^+)_{1^{--}}\rangle|(b\bar{b})\rangle, \quad (8)$$

$$|\Upsilon(1^3D_2)\rangle = |(1_H^- \otimes 2_l^+)_{2^{--}}\rangle|(b\bar{b})\rangle, \quad (9)$$

$$|\Upsilon(1^3D_3)\rangle = |(1_H^- \otimes 2_l^+)_{3^{--}}\rangle|(b\bar{b})\rangle, \quad (10)$$

where the flavor wave function is defined as $|(b\bar{b})\rangle \equiv \frac{1}{\sqrt{2}}(|\bar{b}b\rangle + |b\bar{b}\rangle)$. The superscripts + and - inside the parentheses denote the positive and negative parity of the corresponding parts, respectively, while the superscripts -- and subscripts 0, 1, 2, 3 outside the parentheses correspond to the quantum numbers PC and J of J^{PC} of the bottomonium. The subscripts H and l are used to distinguish the heavy and light spins of a bottomonia. Here, the spin wave functions reflect the C parity of the bottomonia, i.e., $C = (-1)^{S_H+S_l}$.

We also need the spin structures of the light mesons, i.e.,

$$|\pi^+\rangle = |(0_H^+ \otimes 0_l^-)_{0^{++}}\rangle|(u\bar{d})\rangle, \quad (11)$$

$$|\pi^0\rangle = |(0_H^+ \otimes 0_l^-)_{0^{++}}\rangle|\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})\rangle, \quad (12)$$

$$|\pi^-\rangle = |(0_H^+ \otimes 0_l^-)_{0^{++}}\rangle|-(d\bar{u})\rangle, \quad (13)$$

$$|\rho^+\rangle = |(0_H^+ \otimes 1_l^-)_{1^{+-}}\rangle|(u\bar{d})\rangle, \quad (14)$$

$$|\rho^0\rangle = |(0_H^+ \otimes 1_l^-)_{1^{+-}}\rangle|\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})\rangle, \quad (15)$$

$$|\rho^-\rangle = |(0_H^+ \otimes 1_l^-)_{1^{+-}}\rangle|-(d\bar{u})\rangle, \quad (16)$$

$$|\eta\rangle = |(0_H^+ \otimes 0_l^-)_{0^{++}}\rangle|\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u})\rangle, \quad (17)$$

$$|\omega\rangle = |(0_H^+ \otimes 1_l^-)_{1^{+-}}\rangle|\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u})\rangle, \quad (18)$$

$$|\sigma\rangle = |(0_H^+ \otimes 0_l^+)_{0^{++}}\rangle|\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u})\rangle, \quad (19)$$

The orthogonalization of the spin wave functions are defined as

$$\langle (a_H \otimes b_L)_J^{pc} | (c_H \otimes d_L)_{J'}^{p'c'} \rangle = \delta_{ac} \delta_{bd} \delta_{JJ'} \delta_{pp'} \delta_{cc'}, \quad (20)$$

where the superscripts $p^{(\prime)}$ and $c^{(\prime)}$ represent the parity and C parity, respectively. This formula reflects the conservation of the parity, C parity, the total angular momentum, heavy spin, and light spin.

In addition, the orthogonalization of the flavor wave func-

tions leads to

$$\langle (\bar{b}q_i)(b\bar{q}_m)(\bar{b}q_j)(b\bar{q}_n) \rangle = \delta_{ij}\delta_{mn},$$

$$\langle (b\bar{q}_i)(\bar{b}q_m)(\bar{b}q_j)(b\bar{q}_n) \rangle = 0,$$

$$\langle (b\bar{q}_i)(\bar{b}q_m)(b\bar{q}_j)(\bar{b}q_n) \rangle = \delta_{ij}\delta_{mn},$$

$$\langle (\bar{b}q_i)(b\bar{q}_m)(b\bar{q}_j)(\bar{b}q_n) \rangle = 0,$$

where q_i, q_j, q_m and q_n can be u or d quark. We need to specify that the position ordering of the \bar{b} and b , q_i and \bar{q}_m cannot be interchanged. This definition guarantees the orthogonalization of their total wave functions. Moreover, the above definition guarantees that $|B_{(1,2)}^0 \bar{B}^{(*)0}\rangle$ and $|B^{(*)0} \bar{B}_{(1,2)}^0\rangle$ are different physical states.

The effective strong decay Hamiltonian H_{eff} conserves the heavy spin, light spin, isospin, parity, C parity and G -parity separately, which can be decomposed into the spatial and flavor parts,

$$H_{eff} = H_{eff}^{spatial} \otimes H_{eff}^{flavor}, \quad (21)$$

For the decays $B_{(1,2)} \bar{B}^{(*)} \rightarrow (b\bar{b}) + \text{light meson}$, the transition matrix elements related to the flavor wave functions can be written as

$$\langle q_i \bar{q}_m | \langle \bar{b}b | H_{eff}^{flavor} | (\bar{b}q_j)(b\bar{q}_n) \rangle = \delta_{ij}\delta_{mn},$$

$$\langle q_i \bar{q}_m | \langle b\bar{b} | H_{eff}^{flavor} | (\bar{b}q_j)(b\bar{q}_n) \rangle = 0,$$

$$\langle \bar{q}_i q_m | \langle \bar{b}b | H_{eff}^{flavor} | (b\bar{q}_j)(\bar{b}q_n) \rangle = \delta_{ij}\delta_{mn},$$

$$\langle \bar{q}_i q_m | \langle b\bar{b} | H_{eff}^{flavor} | (b\bar{q}_j)(\bar{b}q_n) \rangle = 0.$$

To calculate the strong decays, we also introduce the rearranged spin structure of the final state. Its general expression is

$$\begin{aligned} & |Bottomionia\rangle \otimes |\text{light meson}\rangle \\ &= [(\bar{b}b)_g \otimes L]_K \otimes [Q]_J |(\bar{b}b)\rangle | (q_i \bar{q}_j) \rangle \\ &= \sum_{h=|L-Q|}^{L+Q} \mathcal{D}_{g,h}^{s,L,K,J} \left[\left[(\bar{b}b)_g \otimes [L \otimes Q]_h \right]_J \right] |(\bar{b}b)\rangle | (q_i \bar{q}_j) \rangle \end{aligned} \quad (22)$$

where the indices b, \bar{b} and q_i, \bar{q}_j in the square brackets denote the corresponding spin wave functions. And g and L denote the heavy and light spin of the bottomonium, respectively. We collect the coefficients $\mathcal{D}_{g,h}^{s,L,K,J}$ in Table II.

For the decays $(b\bar{b}) \rightarrow B_{(1,2)} \bar{B}^{(*)} + \text{light meson}$, the transition matrix elements relevant to the flavor wave functions read as

$$\langle q_i \bar{q}_j | \langle (\bar{b}q_m)(b\bar{q}_n) | H_{eff}^{flavor} | b\bar{b} \rangle = 0$$

$$\langle q_i \bar{q}_j | \langle (\bar{b}q_m)(b\bar{q}_n) | H_{eff}^{flavor} | \bar{b}b \rangle = \delta_{im}\delta_{jn} + \delta_{ij}\delta_{mn}$$

$$\langle q_i \bar{q}_j | \langle (b\bar{q}_m)(\bar{b}q_n) | H_{eff}^{flavor} | b\bar{b} \rangle = \delta_{im}\delta_{jn} + \delta_{ij}\delta_{mn}$$

$$\langle q_i \bar{q}_j | \langle (b\bar{q}_m)(\bar{b}q_n) | H_{eff}^{flavor} | \bar{b}b \rangle = 0$$

$$\langle \bar{q}_i q_j | \langle (\bar{b}q_m)(b\bar{q}_n) | H_{eff}^{flavor} | b\bar{b} \rangle = 0$$

$$\langle \bar{q}_i q_j | \langle (\bar{b}q_m)(b\bar{q}_n) | H_{eff}^{flavor} | \bar{b}b \rangle = \delta_{im}\delta_{jn} + \delta_{ij}\delta_{mn}$$

$$\langle \bar{q}_i q_j | \langle (b\bar{q}_m)(\bar{b}q_n) | H_{eff}^{flavor} | b\bar{b} \rangle = \delta_{im}\delta_{jn} + \delta_{ij}\delta_{mn}$$

$$\langle \bar{q}_i q_j | \langle (b\bar{q}_m)(\bar{b}q_n) | H_{eff}^{flavor} | \bar{b}b \rangle = 0.$$

In the decays $(b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}$, the final states need to be decomposed in the similar way,

$$\begin{aligned} & |B_{1,2}\bar{B}^{(*)}\rangle \otimes |\text{light meson}\rangle \\ &= [|\bar{b}\rangle \otimes (q \otimes 1)_s]_K \otimes [b \otimes \bar{q}]_L]_J \otimes (0_H^+ \otimes Q_L^\pm) \\ &= \sum_{g=0}^1 \sum_{m=0}^1 \sum_{h=|s-\frac{1}{2}|}^{s+\frac{1}{2}} \sum_{h_0=|h-Q|}^{h+Q} \mathcal{E}_{g,m,h,h_0}^{s,L,K,J,Q,J_0} \\ & \quad \times [|\bar{b}b\rangle_g \otimes [(q\bar{q})_m \otimes 1]_h \otimes Q]_{h_0} \}_{J_0}, \end{aligned}$$

and

$$\begin{aligned} & |B^{(*)}\bar{B}_{1,2}\rangle \otimes |\text{light meson}\rangle \\ &= [|\bar{b} \otimes \bar{q}]_L \otimes [\bar{b} \otimes (q \otimes 1)_s]_K]_J \otimes (0_H^+ \otimes Q_L^\pm) \\ &= \sum_{g=0}^1 \sum_{m=0}^1 \sum_{h=|s-\frac{1}{2}|}^{s+\frac{1}{2}} \sum_{h_0=|h-Q|}^{h+Q} \mathcal{F}_{g,m,h,h_0}^{s,L,K,J,Q,J_0} \\ & \quad \times [|\bar{b}b\rangle_g \otimes [(q\bar{q})_m \otimes 1]_h \otimes Q]_{h_0} \}_{J_0}, \end{aligned}$$

which will be applied in the following calculation.

For the decays $B_{(1,2)}\bar{B}^{(*)} \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}$, the transition matrix elements are

$$\begin{aligned} & \langle q_i \bar{q}_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (b\bar{q}_k)(\bar{b}q_l) \rangle = 0, \\ & \langle q_i \bar{q}_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (\bar{b}q_k)(\bar{b}q_l) \rangle \\ &= \delta_{in}\delta_{jm}\delta_{kl} + \delta_{il}\delta_{jk}\delta_{mn} + \delta_{ij}\delta_{ml}\delta_{nk} + \delta_{ij}\delta_{mn}\delta_{kl} + \delta_{in}\delta_{jk}\delta_{ml} \\ & \quad + \delta_{il}\delta_{jm}\delta_{nk}, \\ & \langle q_i \bar{q}_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (b\bar{q}_k)(\bar{b}q_l) \rangle \\ &= \delta_{im}\delta_{jn}\delta_{kl} + \delta_{ik}\delta_{jl}\delta_{mn} + \delta_{ij}\delta_{nk}\delta_{ml} + \delta_{ij}\delta_{mn}\delta_{kl} + \delta_{im}\delta_{jl}\delta_{nk} \\ & \quad + \delta_{ik}\delta_{jn}\delta_{ml}, \\ & \langle q_i \bar{q}_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | \bar{b}b \rangle = 0, \\ & \langle \bar{q}_i q_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (b\bar{q}_k)(\bar{b}q_l) \rangle = 0, \\ & \langle \bar{q}_i q_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (\bar{b}q_k)(\bar{b}q_l) \rangle \\ &= \delta_{jm}\delta_{in}\delta_{kl} + \delta_{jk}\delta_{il}\delta_{mn} + \delta_{ij}\delta_{ml}\delta_{nk} + \delta_{ij}\delta_{mn}\delta_{kl} + \delta_{jm}\delta_{il}\delta_{nk} \\ & \quad + \delta_{jk}\delta_{in}\delta_{ml}, \\ & \langle \bar{q}_i q_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (b\bar{q}_k)(\bar{b}q_l) \rangle \\ &= \delta_{jm}\delta_{in}\delta_{kl} + \delta_{jk}\delta_{il}\delta_{mn} + \delta_{ij}\delta_{nk}\delta_{ml} + \delta_{ij}\delta_{mn}\delta_{kl} + \delta_{jm}\delta_{il}\delta_{nk} \\ & \quad + \delta_{jk}\delta_{in}\delta_{ml}, \\ & \langle \bar{q}_i q_j | \langle (\bar{b}q_m)(\bar{b}q_n) | H_{eff}^{flavor} | (\bar{b}q_k)(\bar{b}q_l) \rangle = 0. \end{aligned}$$

III. NUMERICAL RESULTS

With the help of heavy quark symmetry, the conservations of parity, C-parity and G-parity, we are ready to discuss the strong decays of the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems, which

can be categorized into three groups:

$$\begin{aligned} & B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) \rightarrow (b\bar{b}) + \text{light meson}, \\ & (b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) + \text{light meson}, \\ & B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) + \text{light meson} \\ & \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) + \text{light meson}. \end{aligned}$$

We collect the typical decay ratios in Tables V-X, where the ratios in the brackets are the results considering the contribution from the phase space factors. We need to specify that in this work we do not introduce any dynamics model for the strong decays. Generally the decay width of a specific decay channel is proportional to the spatial matrix elements which are related to its spatial wave functions. Only if the initial systems and final states belong to the same heavy spin multiplet, the spatial matrix elements of these strong decays are the same, which leads to quite simple ratios between their decay widths, as we have discussed in our former work [52, 53]. Since the masses of B_0 meson and D-wave bottomonia are still absent experimentally, we ignore the contribution of the phase space factors when calculating the corresponding ratios. In the following, we present the numerical results.

A. $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) \rightarrow (b\bar{b}) + \text{light meson}$

There are six $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems with $J^{PC} = 1^{--}$. Except $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$ with isospin $I = 1$, all the other systems with $I = 1$ can decay into χ_{bJ} ($J = 0, 1, 2$) via the ρ emission. These decays are governed by the spin configuration $(1_H^- \otimes 1_l^+)|_{J=1}^-$. Since $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$ contains the configuration $(0_H^- \otimes 1_l^+)|_{J=1}^-$ only, the isovector decay mode $\chi_{bJ\rho}$ is not allowed in the heavy quark limit.

All the isovector states of the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems can decay into $h_b\pi$. This decay mode depends on the spin configuration $(0_H^- \otimes 1_l^+)|_{J=1}^-$. And their decay widths are proportional to the parameter H_π^{11} as listed in Table III, which is defined as $H_\pi^{ij} \propto \langle 0, i | H_{eff}(\pi) | j \rangle$. And the indices i and j represent the light spin of the final and initial hadron, respectively, where the $H_{eff}(\pi)$ denotes the effective Hamiltonian for the one-pion decay.

We also calculate the strong decay ratios of the isovector $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems with $J^{PC} = 1^{--}$, which are listed in Table V. Except the isovector $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$ system, the remaining five systems have the same ratio $\Gamma(\chi_{b0\rho}) : \Gamma(\chi_{b1\rho}) : \Gamma(\chi_{b2\rho}) = 4 : 3 : 5$ without considering the contribution of the phase space factor.

The isovector states of $\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$ and $B^*\bar{B}^*$ with $J = 1^{+-}$ were considered as the most possible candidates of $Z_b(10610)$ and $Z_b(10650)$, respectively [54]. Both of them can decay into $\eta_{b\rho}$ and $\eta_{b2\rho}$ via the spin configuration $(0_H^- \otimes 1_l^+)|_{J=1}^+$ as shown in Table III. They can also decay into $\Upsilon(1^3S_1)$ via the one-pion emission through the spin configuration $(1_H^- \otimes 1_l^+)|_{J=1}^+$. Their one-pion decay mode $\Upsilon(1^3D_1)\pi$ depends on the spin configuration $(1_H^- \otimes 2_l^-)|_{J=1}^+$. According to the spin structure of $\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$ and $B^*\bar{B}^*$ with $J = 1^{+-}$,

TABLE II: The coefficient $\mathcal{D}_{g,h}^{L,K,J}$ in Eq. (22) corresponding to different combinations of $[g, h]$.

	$J = 0$		$J = 1$				$J = 2$			
	[0, 0]	[1, 1]	[0, 1]	[1, 0]	[1, 1]	[1, 2]	[0, 2]	[1, 1]	[1, 2]	[1, 3]
$ \eta_b(1^1S_0)\pi/\eta/\sigma\rangle$	1	0	—	—	—	—	—	—	—	—
$ \Upsilon(1^3S_1)\pi/\eta/\sigma\rangle$	—	—	0	1	0	0	—	—	—	—
$ h_b(1^1P_1)\pi/\eta/\sigma\rangle$	—	—	1	0	0	0	—	—	—	—
$ \chi_{b0}(1^3P_0)\pi/\eta/\sigma\rangle$	0	1	—	—	—	—	—	—	—	—
$ \chi_{b1}(1^3P_1)\pi/\eta/\sigma\rangle$	—	—	0	0	1	0	—	—	—	—
$ \chi_{b2}(1^3P_2)\pi/\eta/\sigma\rangle$	—	—	—	—	—	—	0	1	0	0
$ \eta_{b2}(1^1D_2)\pi/\eta/\sigma\rangle$	—	—	—	—	—	—	1	0	0	0
$ \Upsilon(1^3D_1)\pi/\eta/\sigma\rangle$	—	—	0	0	0	1	—	—	—	—
$ \Upsilon(1^3D_2)\pi/\eta/\sigma\rangle$	—	—	—	—	—	—	0	0	1	0
$ \eta_b(1^1S_0)\rho/\omega\rangle$	—	—	1	0	0	0	—	—	—	—
$ \Upsilon(1^3S_1)\rho/\omega\rangle$	0	1	0	0	1	0	0	1	0	0
$ h_b(1^1P_1)\rho/\omega\rangle$	1	0	1	0	0	0	1	0	0	0
$ \chi_{b0}(1^3P_0)\rho/\omega\rangle$	—	—	0	$\frac{1}{3}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{5}}{3}$	—	—	—	—
$ \chi_{b1}(1^3P_1)\rho/\omega\rangle$	0	1	0	$-\frac{\sqrt{3}}{3}$	$\frac{1}{2}$	$\frac{\sqrt{15}}{6}$	0	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
$ \chi_{b2}(1^3P_2)\rho/\omega\rangle$	—	—	0	$\frac{\sqrt{5}}{3}$	$\frac{\sqrt{15}}{6}$	$\frac{1}{6}$	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0
$ \eta_{b2}(1^1D_2)\rho/\omega\rangle$	—	—	1	0	0	0	1	0	0	0
$ \Upsilon(1^3D_1)\rho/\omega\rangle$	0	1	0	0	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0	$\frac{1}{10}$	$-\frac{\sqrt{15}}{10}$	$\frac{\sqrt{21}}{5}$
$ \Upsilon(1^3D_2)\rho/\omega\rangle$	—	—	0	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{\sqrt{15}}{10}$	$\frac{5}{6}$	$\frac{\sqrt{35}}{15}$

we conclude that the decay mode $\Upsilon(1^3D_1)\pi$ of $Z_b(10610)$ and $Z_b(10650)$ are strongly suppressed due to the heavy quark symmetry as shown in Table III.

For the $\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$ system with $J = 1^{++}$, its isovector states have decay modes $\Upsilon(1^3S_1)\rho$, $\Upsilon(1^3D_1)\rho$ and $\Upsilon(1^3D_2)\rho$, where the spin configuration $(1_H^- \otimes 1_l^-)|_{J=1}^{++}$ is dominant. The branching ratio of the isovector states relevant to the $\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$ system with $J = 1^{++}$ is $\Gamma(\Upsilon(1^3D_1)\rho) : \Gamma(\Upsilon(1^3D_2)\rho) = 1 : 3$ as listed in Table V.

From Table III, we notice that except $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* - B^*\bar{B}_1')$ the remaining the isovector states of the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems with $J^{PC} = 1^{-+}$ can decay into $\chi_{b1}\pi$ and $h_b\rho$, which depend on the spin configuration $(1_H^- \otimes 1_l^+)|_{J=1}^{+-}$ and $(0_H^- \otimes 1_l^+)|_{J=1}^{+-}$ respectively. However, $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* - B^*\bar{B}_1')$ only has the spin configurations $(0_H^- \otimes 1_l^+)|_{J=1}^{+-}$ and $(1_H^- \otimes 0_l^+)|_{J=1}^{+-}$. Thus, the decays of these isovector states into $\chi_{b1}\pi$ are suppressed due to the conservations of heavy spin, light spin and C -parity.

Except $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$, the isoscalar states relevant to the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems with $J^{PC} = 1^{--}$ have the allowed decay modes $h_b\eta$, $\chi_{b0}\omega$, $\chi_{b1}\omega$ and $\chi_{b2}\omega$, which is similar to these decays of its isovector partners into $h_b\pi$ and $\chi_{bJ}\rho$, where $h_b\pi$ is related to the spin configuration $(0_H^- \otimes 1_l^+)|_{J=1}^{+-}$ and $\chi_{bJ}\rho$ is due to the contribution of the spin configuration $(1_H^- \otimes 1_l^+)|_{J=1}^{+-}$. The decay mode $h_b\eta$ of the isoscalar states rel-

evant to the $\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$ system is also allowed through the configuration $(0_H^- \otimes 1_l^+)|_{J=1}^{+-}$, but its decay into $\chi_{bJ}\omega$ is suppressed in the heavy quark limit. As shown in Table III, these isoscalar partners relevant to the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems with $J^{PC} = 1^{--}$ cannot decay into $\Upsilon\sigma$ and $\Upsilon(1^3D_1)\sigma$. This phenomena can be understood well since the $\Upsilon(1^3S_1)\sigma$ and $\Upsilon(1^3D_1)\sigma$ decay modes are governed by the spin configuration $(1_H^- \otimes 0_l^+)|_{J=1}^{--}$ and $(1_H^- \otimes 2_l^+)|_{J=1}^{--}$, respectively. These two spin configurations do not appear in the spin structures of the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*)$ systems. Therefore their decays into $\Upsilon(1^3S_1)\sigma$ and $\Upsilon(1^3D_1)\sigma$ are strongly suppressed due to the heavy quark symmetry. The branching ratios of them into $\chi_{bJ}\omega$ is $\Gamma(\chi_{b0}\rho) : \Gamma(\chi_{b1}\rho) : \Gamma(\chi_{b2}\rho) = 4 : 3 : 5$ if ignoring the phase space difference. The other suppressed channels in Table III are similar to the corresponding decay channels of their isovector partners.

As listed in Table IV, the isovector states of the $B^*\bar{B}^*$ system with $J^{PC} = 2^{++}$ can decay into $\Upsilon(1^3S_1)\rho$ and $\Upsilon(1^3D_J)\rho$ through the spin configuration $(1_H^- \otimes 1_l^-)|_{J=1}^{++}$. But its decay mode $\eta_{b2}\pi$ is suppressed which depends on the spin configuration $(0_H^- \otimes 2_l^-)|_{J=1}^{++}$. However, $B^*\bar{B}^*$ doesn't contain this spin configuration. The typical ratio of $B^*\bar{B}^*$ decays into $\Upsilon(1^3D_J)\rho$ is $\Gamma(\Upsilon(1^3D_1)\rho) : \Gamma(\Upsilon(1^3D_2)\rho) : \Gamma(\Upsilon(1^3D_3)\rho) = 1 : 15 : 84$.

There are four $B_{(1,2)}\bar{B}^{(*)}$ systems with $J^{PC} = 2^{--}$. Their isovector partners can decay into χ_{b1} and χ_{b2} through the emission of the ρ meson, where the spin configuration $(1_H^- \otimes$

TABLE III: The typical relations between the decay widths $\Gamma(B_{(1,2)}\bar{B}^{(*)} \rightarrow (b\bar{b}) + \text{light meson})$ and the reduced matrix elements $H_\alpha^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$, where the indices i and j denote the light spin of the final and initial hadron respectively, and Q is the angular momentum of the final light meson.

$I^G(J^{PC})$	Initial state	Final state			
		$h_b\pi$	$\chi_{b0}\rho$	$\chi_{b1}\rho$	$\chi_{b2}\rho$
$1^+(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	$\frac{\sqrt{6}}{3}H_\pi^{11}$	$-\frac{\sqrt{2}}{3}H_\rho^{11}$	$-\frac{\sqrt{6}}{6}H_\rho^{11}$	$-\frac{\sqrt{10}}{6}H_\rho^{11}$
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	$-\frac{\sqrt{6}}{3}H_\pi^{11}$	$-\frac{\sqrt{2}}{3}H_\rho^{11}$	$-\frac{\sqrt{6}}{6}H_\rho^{11}$	$-\frac{\sqrt{10}}{6}H_\rho^{11}$
	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$	$-\frac{\sqrt{3}}{3}H_\pi^{11}$	$-\frac{1}{3}H_\rho^{11}$	$\frac{\sqrt{3}}{6}H_\rho^{11}$	$\frac{\sqrt{5}}{6}H_\rho^{11}$
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	$\frac{2\sqrt{3}}{3}H_\pi^{11}$	0	0	0
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$-\frac{\sqrt{6}}{6}H_\pi^{11}$	$-\frac{\sqrt{2}}{2}H_\rho^{11}$	$\frac{\sqrt{6}}{4}H_\rho^{11}$	$\frac{\sqrt{10}}{4}H_\rho^{11}$
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	$-\frac{\sqrt{30}}{6}H_\pi^{11}$	$\frac{\sqrt{10}}{6}H_\rho^{11}$	$-\frac{\sqrt{30}}{12}H_\rho^{11}$	$-\frac{5\sqrt{2}}{12}H_\rho^{11}$
$I^G(J^{PC})$	Initial state	Final state			
		$\Upsilon(1^3S_1)\pi$	$\Upsilon(1^3D_1)\pi$	$\eta_b\rho$	$\eta_{b2}\rho$
$1^+(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	$-H_\pi^{00}$	0	H_ρ^{01}	H_ρ^{21}
	$B^*\bar{B}^*$	H_π^{00}	0	H_ρ^{01}	H_ρ^{21}
$I^G(J^{PC})$	Initial state	Final state			
		$\Upsilon(1^3S_1)\rho$	$\Upsilon(1^3D_1)\rho$	$\Upsilon(1^3D_2)\rho$	
$1^-(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	$\sqrt{2}H_\rho^{01}$	$-\frac{\sqrt{2}}{2}H_\rho^{21}$	$\frac{\sqrt{6}}{2}H_\rho^{21}$	
$I^G(J^{PC})$	Initial state	Final state			
		$\chi_{b1}\pi$	$h_b\rho$		
$1^-(1^{+-})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$	$\frac{2\sqrt{3}}{3}H_\pi^{11}$	$\frac{\sqrt{3}}{3}H_\rho^{11}$		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	$\frac{2\sqrt{3}}{3}H_\pi^{11}$	$-\frac{\sqrt{3}}{3}H_\rho^{11}$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$	$-\frac{\sqrt{6}}{6}H_\pi^{11}$	$\frac{\sqrt{6}}{3}H_\rho^{11}$		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	0	$\frac{\sqrt{6}}{3}H_\rho^{11}$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$-\frac{\sqrt{3}}{2}H_\pi^{11}$	$\frac{\sqrt{3}}{3}H_\rho^{11}$		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	$\frac{\sqrt{15}}{6}H_\pi^{11}$	$\frac{\sqrt{15}}{3}H_\rho^{11}$		
$I^G(J^{PC})$	Initial state	Final state			
		$\Upsilon(1^3S_1)\sigma$	$\Upsilon(1^3D_1)\sigma$	$h_b\eta$	$\chi_{b0}\omega$
$0^-(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	0	0	$\frac{\sqrt{6}}{3}H_\eta^{11}$	$-\frac{\sqrt{2}}{3}H_\omega^{11}$
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	0	0	$-\frac{\sqrt{6}}{3}H_\eta^{11}$	$-\frac{\sqrt{2}}{3}H_\omega^{11}$
	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$	0	0	$-\frac{\sqrt{3}}{3}H_\eta^{11}$	$-\frac{1}{3}H_\omega^{11}$
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	0	0	$\frac{2\sqrt{3}}{3}H_\eta^{11}$	0
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	0	0	$-\frac{\sqrt{6}}{6}H_\eta^{11}$	$-\frac{\sqrt{2}}{2}H_\omega^{11}$
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	0	0	$-\frac{\sqrt{30}}{6}H_\eta^{11}$	$\frac{\sqrt{10}}{6}H_\omega^{11}$
$I^G(J^{PC})$	Initial state	Final state			
		$h_b\sigma$	$\Upsilon(1^3S_1)\eta$	$\Upsilon(1^3D_1)\eta$	$\eta_b\omega$
$0^-(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	H_σ^{11}	$-H_\pi^{00}$	0	H_ω^{01}
	$B^*\bar{B}^*$	H_σ^{11}	H_π^{00}	0	H_ω^{01}
$I^G(J^{PC})$	Initial state	Final state			
		$\chi_{b1}\sigma$	$\Upsilon(1^3S_1)\omega$	$\Upsilon(1^3D_1)\omega$	$\Upsilon(1^3D_2)\omega$
$0^+(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	$\sqrt{2}H_\sigma^{01}$	$\sqrt{2}H_\omega^{01}$	$-\frac{\sqrt{2}}{2}H_\omega^{21}$	$\frac{\sqrt{6}}{2}H_\omega^{21}$
$I^G(J^{PC})$	Initial state	Final state			
		$\chi_{b1}\eta$	$h_b\omega$		
$0^+(1^{+-})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$	$\frac{2\sqrt{3}}{3}H_\eta^{11}$	$\frac{\sqrt{3}}{3}H_\omega^{11}$		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	$\frac{2\sqrt{3}}{3}H_\eta^{11}$	$-\frac{\sqrt{3}}{3}H_\omega^{11}$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$	$-\frac{\sqrt{6}}{6}H_\eta^{11}$	$\frac{\sqrt{6}}{3}H_\omega^{11}$		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	0	$\frac{\sqrt{6}}{3}H_\omega^{11}$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$-\frac{\sqrt{3}}{2}H_\eta^{11}$	$\frac{\sqrt{3}}{3}H_\omega^{11}$		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	$\frac{\sqrt{15}}{6}H_\eta^{11}$	$\frac{\sqrt{15}}{3}H_\omega^{11}$		

TABLE IV: The typical relations between the decay widths $\Gamma(B_{(1,2)}\bar{B}^{(*)} \rightarrow (b\bar{b}) + \text{light meson})$ and the reduced matrix elements $H_{\alpha}^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$, where the indices i and j denote the light spin of the final and initial hadron respectively, and Q means the angular momentum of the final light meson.

$I^G(J^{PC})$	Initial state	Final state				
		$\eta_{b2}\pi$	$\Upsilon(1^3S_1)\rho$	$\Upsilon(1^3D_1)\rho$	$\Upsilon(1^3D_2)\rho$	$\Upsilon(1^3D_3)\rho$
$1^-(2^{++})$	$B^*\bar{B}^*$	0	$\sqrt{2}H_{\rho}^{01}$	$\frac{\sqrt{3}}{10}H_{\rho}^{21}$	$-\frac{\sqrt{30}}{10}H_{\rho}^{21}$	$\frac{\sqrt{42}}{5}H_{\rho}^{21}$
$I^G(J^{PC})$	Initial state	Final state				
		$\chi_{b1}\rho$	$\chi_{b2}\rho$			
	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* - B^*\bar{B}_1')$	$-\frac{\sqrt{3}}{3}H_{\rho}^{11}$	H_{ρ}^{11}			
$1^+(2^{--})$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$\frac{\sqrt{6}}{12}H_{\rho}^{11}$	$-\frac{\sqrt{2}}{4}H_{\rho}^{11}$			
	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)$	$-\frac{1}{2}H_{\rho}^{11}$	$\frac{\sqrt{3}}{2}H_{\rho}^{11}$			
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	$-\frac{\sqrt{6}}{4}H_{\rho}^{11}$	$\frac{3\sqrt{2}}{4}H_{\rho}^{11}$			
$I^G(J^{PC})$	Initial state	Final state				
		$\chi_{b2}\pi$	$h_b\rho$			
	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$	$-\frac{2\sqrt{6}}{3}H_{\pi}^{11}$	0			
$1^-(2^{-+})$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$-\frac{\sqrt{3}}{6}H_{\pi}^{11}$	0			
	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)$	$-\frac{\sqrt{2}}{2}H_{\pi}^{11}$	0			
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	$-\frac{\sqrt{3}}{2}H_{\pi}^{11}$	0			
$I^G(J^{PC})$	Initial state	Final state				
		$\eta_b\pi$	$\Upsilon(1^3S_1)\rho$	$\Upsilon(1^3D_1)\rho$		
	$B\bar{B}$	$\frac{\sqrt{2}}{2}H_{\pi}^{00}$	$\frac{\sqrt{6}}{2}H_{\rho}^{01}$	$\frac{\sqrt{6}}{20}H_{\rho}^{21}$		
$1^-(0^{++})$	$B^*\bar{B}^*$	$\frac{\sqrt{3}}{2}H_{\pi}^{00}$	$-\frac{\sqrt{2}}{2}H_{\rho}^{01}$	$-\frac{\sqrt{2}}{20}H_{\rho}^{21}$		
$I^G(J^{PC})$	Initial state	Final state	$I^G(J^{PC})$	Initial state	Final state	
		$\chi_{b1}\pi$	$h_b\rho$		$\chi_{b1}\rho$	
	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)$	$\sqrt{2}H_{\pi}^{11}$	0	$\frac{1}{\sqrt{2}}(B_0\bar{B} - B\bar{B}_0)$	H_{ρ}^{11}	
$1^-(0^{-+})$	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$	$-\frac{\sqrt{6}}{3}H_{\pi}^{11}$	0	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* - B^*\bar{B}_1')$	$-\frac{\sqrt{3}}{3}H_{\rho}^{11}$	
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$\frac{2\sqrt{3}}{3}H_{\pi}^{11}$	0	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$-\frac{2\sqrt{6}}{3}H_{\rho}^{11}$	
$I^G(J^{PC})$	Initial state	Final state				
		$\chi_{b2}\sigma$	$\eta_{b2}\eta$	$\Upsilon(1^3S_1)\omega$	$\Upsilon(1^3D_1)\omega$	$\Upsilon(1^3D_2)\omega$
$0^+(2^{++})$	$B^*\bar{B}^*$	$\sqrt{2}H_{\sigma}^{01}$	0	$\sqrt{2}H_{\omega}^{01}$	$\frac{\sqrt{3}}{10}H_{\omega}^{21}$	$-\frac{\sqrt{30}}{10}H_{\omega}^{21}$
		$\Upsilon(1^3D_2)\sigma$	$\chi_{b1}\omega$	$\chi_{b2}\omega$		
	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* - B^*\bar{B}_1')$	0	$-\frac{\sqrt{3}}{3}H_{\omega}^{11}$	H_{ω}^{11}		
$0^-(2^{--})$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	0	$\frac{\sqrt{6}}{12}H_{\omega}^{11}$	$-\frac{\sqrt{2}}{4}H_{\omega}^{11}$		
	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)$	0	$-\frac{1}{2}H_{\omega}^{11}$	$\frac{\sqrt{3}}{2}H_{\omega}^{11}$		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	0	$-\frac{\sqrt{6}}{4}H_{\omega}^{11}$	$\frac{3\sqrt{2}}{4}H_{\omega}^{11}$		
$I^G(J^{PC})$	Initial state	Final state				
		$\eta_{b2}\sigma$	$\chi_{b2}\eta$	$h_b\omega$		
	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$	0	$-\frac{2\sqrt{6}}{3}H_{\eta}^{11}$	0		
$0^+(2^{-+})$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	0	$-\frac{\sqrt{3}}{6}H_{\eta}^{11}$	0		
	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)$	0	$-\frac{\sqrt{2}}{2}H_{\eta}^{11}$	0		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	0	$-\frac{\sqrt{3}}{2}H_{\eta}^{11}$	0		
$I^G(J^{PC})$	Initial state	Final state				
		$\chi_{b0}\sigma$	$\eta_b\eta$	$\Upsilon(1^3S_1)\omega$	$\Upsilon(1^3D_1)\omega$	
	$B\bar{B}$	$\frac{\sqrt{6}}{2}H_{\sigma}^{11}$	$\frac{\sqrt{2}}{2}H_{\eta}^{00}$	$\frac{\sqrt{6}}{2}H_{\omega}^{01}$	$\frac{\sqrt{6}}{20}H_{\omega}^{21}$	
$0^*(0^{++})$	$B^*\bar{B}^*$	$-\frac{\sqrt{2}}{2}H_{\sigma}^{11}$	$\frac{\sqrt{3}}{2}H_{\eta}^{00}$	$-\frac{\sqrt{2}}{2}H_{\omega}^{01}$	$-\frac{\sqrt{2}}{20}H_{\omega}^{21}$	
$I^G(J^{PC})$	Initial state	Final state	$I^G(J^{PC})$	Initial state	Final state	
		$\eta_b\sigma$	$\chi_{b1}\eta$	$h_b\omega$	$\chi_{b1}\omega$	
	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)$	0	$\sqrt{2}H_{\eta}^{11}$	0	$\frac{1}{\sqrt{2}}(B_0\bar{B} - B\bar{B}_0)$	H_{ω}^{11}
$0^*(0^{-+})$	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* + B^*\bar{B}_1')$	0	$-\frac{\sqrt{6}}{3}H_{\eta}^{11}$	0	$\frac{1}{\sqrt{2}}(B_1'\bar{B}^* - B^*\bar{B}_1')$	$-\frac{\sqrt{3}}{3}H_{\omega}^{11}$
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	0	$\frac{2\sqrt{3}}{3}H_{\eta}^{11}$	0	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$-\frac{2\sqrt{6}}{3}H_{\omega}^{11}$

TABLE V: The typical ratios of the $B_{(1,2)}\bar{B}^{(*)} \rightarrow (b\bar{b}) + \text{light meson}$ decay widths.

$I^G(J^{PC})$		Final state
Initial state	$1^+(1^{--})$	$\Gamma(\chi_{b0}\rho) : \Gamma(\chi_{b1}\rho) : \Gamma(\chi_{b2}\rho)$ 4 : 3 : 5
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	4 : 3 : 5 (1.60 : 1 : 1.48)
	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$	4 : 3 : 5 (1.57 : 1 : 1.50)
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	0 : 0 : 0
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	4 : 3 : 5 (1.54 : 1 : 1.52)
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	4 : 3 : 5 (1.53 : 1 : 1.53)
	$1^-(1^{++})$	$\Gamma(\Upsilon(1^3D_1)\rho) : \Gamma(\Upsilon(1^3D_2)\rho)$ 1 : 3
	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	
	$1^+(2^{--})$	$\Gamma(\chi_{b1}\rho) : \Gamma(\chi_{b2}\rho)$ 1 : 3 (1 : 2.73)
Initial state	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	1 : 3 (1 : 2.70)
	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)$	1 : 3 (1 : 2.72)
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	1 : 3 (1 : 2.75)
	$1^-(2^{++})$	$\Gamma(\Upsilon(1^3D_1)\rho) : \Gamma(\Upsilon(1^3D_2)\rho) : \Gamma(\Upsilon(1^3D_3)\rho)$ 1 : 15 : 84
	$B^*\bar{B}^*$	
	$0^-(1^{--})$	$\Gamma(\chi_{b0}\omega) : \Gamma(\chi_{b1}\omega) : \Gamma(\chi_{b2}\omega)$ 4 : 3 : 5
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	4 : 3 : 5 (1.61 : 1 : 1.46)
	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$	4 : 3 : 5 (1.57 : 1 : 1.50)
Initial state	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	0 : 0 : 0
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	4 : 3 : 5 (1.55 : 1 : 1.51)
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	4 : 3 : 5 (1.53 : 1 : 1.52)
	$0^+(1^{++})$	$\Gamma(\Upsilon(1^3D_1)\omega) : \Gamma(\Upsilon(1^3D_2)\omega)$ 1 : 3
	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	
	$0^-(2^{--})$	$\Gamma(\chi_{b1}\omega) : \Gamma(\chi_{b2}\omega)$ 1 : 3 (1 : 2.73)
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	1 : 3 (1 : 2.69)
	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)$	1 : 3 (1 : 2.71)
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	1 : 3 (1 : 2.74)
Initial state	$0^+(2^{++})$	$\Gamma(\Upsilon(1^3D_1)\omega) : \Gamma(\Upsilon(1^3D_2)\omega) : \Gamma(\Upsilon(1^3D_3)\omega)$ 1 : 15 : 84
	$B^*\bar{B}^*$	

$1^+_{J=2}|_{J=2}^{--}$ is dominant. Our result indicates that $\Gamma(\chi_{b1}\rho) : \Gamma(\chi_{b2}\rho) = 1 : 3$ for all four states. When the initial states are the $B_{(1,2)}\bar{B}^{(*)}$ systems with $J^{PC} = 2^{+-}$, their decays into $\chi_{b2}\pi$ depend on the spin configuration $(1^-_H \otimes 1^+_I)|_{J=2}^{+-}$. The decay mode $h_b\rho$ is suppressed as shown in Table IV since these decays are only governed by the spin configuration $(0^-_H \otimes 2^+_I)|_{J=1}^{+-}$. Similar conclusions hold for the three $B_{(1,2)}\bar{B}^{(*)}$ systems with $J^{PC} = 0^{+-}$.

The isoscalar partner of $B^*\bar{B}^*$ system only contain the spin configuration $(1^-_H \otimes 1^+_I)|_{J=2}^{++}$ with $J^{PC} = 2^{++}$. Thus its decay into $\eta_{b2}\eta$ is suppressed. The $\chi_{b1}\omega$ and $\chi_{b2}\omega$ are the allowed decay modes of the isoscalar partners relevant to the $B_{(1,2)}\bar{B}^{(*)}$ systems with $J^{PC} = 2^{--}$, where the $(1^-_H \otimes 1^+_I)|_{J=2}^{--}$ component is dominant. We have the typical ratios $\Gamma(\chi_{b1}\omega) : \Gamma(\chi_{b2}\omega) = 1 : 3$, which is the same as that of their isovector partners. However, the decay mode $\Upsilon(1^3D_2)\sigma$ of all the isoscalar partners of the $B_{(1,2)}\bar{B}^{(*)}$ systems with $J^{PC} = 2^{--}$ is suppressed due to the absence of the spin configuration $(1^-_H \otimes 2^+_I)|_{J=2}^{--}$.

From Table IV, we can see that the decay modes $\eta_{b2}\sigma$ and $h_b\omega$ of the four isoscalar partners relevant to the $B_{(1,2)}\bar{B}^{(*)}$

systems with $J^{PC} = 2^{--}$ are suppressed. These decays are governed by the $(0^-_H \otimes 2^+_I)|_{J=2}^{+-}$ configuration, while the four isoscalar states only contain spin configuration with heavy spin equal to 1. Similar situations occur in the suppressed decay channels $\eta_b\sigma$ and $h_b\omega$ of the isoscalar states relevant to the $B_{(1,2)}\bar{B}^{(*)}$ systems with $J^{PC} = 0^{+-}$ shown in Table IV.

We need to specify that the ratios shown in Tables V and VI are also suitable for the strong decays involved with the higher radially excited bottomonia as long as these decays are kinematically allowed. Since the dominant decay modes of the σ/ρ and ω mesons are 2π and 3π , the above numerical results can be easily extended to discuss the di-pion and tri-pion strong decays.

B. $(b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) + \text{light meson}$

In this subsection, we investigate the production of the hidden beauty molecular/resonant states via the strong decays of the higher radial excitations of bottomonium. Among all the

TABLE VI: The typical ratios $\frac{\Gamma(B_{(1,2)}\bar{B}^{(*)}\rightarrow(b\bar{b})+light\ meson)}{\Gamma(B_{(1,2)}\bar{B}^{(*)}\rightarrow(bb)+light\ meson)}$, where the initial molecular states are different while the final states are the same.

Initial state	$I^G(J^{PC})$	Final state					
		$h_b\pi$	$\chi_{b0}\rho$	$\chi_{b1}\rho$	$\chi_{b2}\rho$		
$1^+(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)$	1 : 2	4 : 0	4 : 0	4 : 0		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)$						
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)$	1 : 5 (1 : 5.09)	9 : 5 (1.62 : 1)	9 : 5 (1.61 : 1)	9 : 5 (1.60 : 1)		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)$						
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)$	1 : 2 (1 : 2.07)	4 : 0	4 : 0	4 : 0		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)$						
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)$	2 : 1 (1.94 : 1)	2 : 9 (1 : 5.45)	2 : 9 (1 : 5.54)	2 : 9 (1 : 5.61)		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)$						
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)$	1 : 1	1 : 1	1 : 1	1 : 1		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)$						
$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)$	2 : 5 (1 : 2.62)	2 : 5 (1 : 3.36)	2 : 5 (1 : 3.44)	2 : 5 (1 : 3.50)			
$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)$							
$0^-(1^{--})$	$\Upsilon(1^3S_1)\sigma$	$\Upsilon(1^3D_1)\sigma$	$h_b\eta$	$\chi_{b0}\omega$	$\chi_{b1}\omega$	$\chi_{b2}\omega$	
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)$	0 : 0	0 : 0	1 : 2	4 : 0	4 : 0	4 : 0
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)$						
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)$	0 : 0	0 : 0	1 : 5 (1 : 5.12)	9 : 5 (1.62 : 1)	9 : 5 (1.61 : 1)	9 : 5 (1.60 : 1)
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)$						
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)$	0 : 0	0 : 0	1 : 2 (1 : 2.10)	4 : 0	4 : 0	4 : 0
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)$						
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)$	0 : 0	0 : 0	2 : 1 (1.92 : 1)	2 : 9 (1 : 5.48)	2 : 9 (1 : 5.58)	2 : 9 (1 : 5.65)
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)$						
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)$	0 : 0	0 : 0	1 : 1	1 : 1	1 : 1	1 : 1
$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)$							
$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)$	0 : 0	0 : 0	2 : 5 (1 : 2.67)	2 : 5 (1 : 3.39)	2 : 5 (1 : 3.47)	2 : 5 (1 : 3.54)	
$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)$							
$1^-(1^{+-})$	$\chi_{b1}\pi$	$h_b\rho$			$\chi_{b1}\eta$	$h_b\omega$	
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)$	16 : 0	1 : 2		$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)$	16 : 0	1 : 2
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)$				$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)$	9 : 5 (1.77 : 1)	1 : 5 (1 : 5.59)	$0^+(1^{+-})$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)$	9 : 5 (1.76 : 1)	1 : 5 (1 : 5.61)
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)$				$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)$		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)$	16 : 0	1 : 2 (1 : 2.56)		$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)$	16 : 0	1 : 2 (1 : 2.58)
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)$				$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)$	2 : 9 (1 : 4.64)	2 : 1 (1.62 : 1)		$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)$	2 : 9 (1 : 4.69)	2 : 1 (1.61 : 1)
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)$				$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)$		
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)$	1 : 1	1 : 1		$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)$	1 : 1	1 : 1
$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)$				$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)$			
$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)$	2 : 5 (1 : 2.62)	2 : 5 (1 : 3.46)		$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)$	2 : 5 (1 : 2.67)	2 : 5 (1 : 3.49)	
$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)$				$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)$			
$1^+(1^{+-})$	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})}{B^*\bar{B}^*}$	$\Upsilon(1^3S_1)\pi$	$\Upsilon(^3D_1)\pi$	$\eta_b\rho$	$\eta_{b2}(1^1D_2)\rho$		
		1 : 1 (1 : 1.03)	0 : 0	1 : 1 (1 : 1.18)	1 : 1		
$0^-(1^{+-})$	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})}{B^*\bar{B}^*}$	$h_b\sigma$	$\Upsilon(1^3S_1)\eta$	$\Upsilon(^3D_1)\eta$	$\eta_b\omega$	$\eta_{b2}(1^1D_2)\omega$	
		1 : 1 (1 : 1.20)	1 : 1 (1 : 1.04)	0 : 0	1 : 1 (1 : 1.19)	1 : 1	

hidden beauty systems considered here, the lowest mass state is $\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$ which is about 10610 MeV. The pion mass is 135 MeV. Thus, we are interested in the decays of the bottomonia with the mass around 10745 MeV or higher radial excited states in the bottomonium family like $\Upsilon(11020)$.

Using the same spin rearrangement scheme approach, we list the typical relations between the strong decay widths $\Gamma((b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B} \text{ or } B^*\bar{B}^*) + light\ meson)$ and its corresponding reduced matrix elements relevant to the light spin. The parameter H_m^{ij} is the reduced matrix element with $H_m^{ij} \propto \langle Q, i | H_{eff}(m) | j \rangle$, where the i and j indices denote the light

spin of the final and initial hadron, respectively. m can be $\pi, \eta, \rho, \omega, \sigma$ meson, while Q is the spin of light meson. These results are collected in Tables VII-VIII. We also calculate the strong decay ratios, which are shown in Tables IX-X.

$$\mathbf{C.} \\ B_{(1,2)}\bar{B}^{(*)}(B\bar{B}^* \text{ or } B^*\bar{B}^*) \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B}^* \text{ or } B^*\bar{B}^*) + light\ meson$$

With the help of the heavy quark symmetry, we further discuss the strong decays between two hidden beauty

TABLE VII: The typical relations between the decay widths $\Gamma((b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson})$ and the reduced matrix elements $H_\alpha^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$, where the indices i and j denote the light spin of the final and initial hadron respectively, and Q denotes the angular momentum of the final light meson.

Initial state		Final state $I^G(J^{PC}) = 1^+(1^{--})$						
$h_b(n^1P_1)$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\pi$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\pi$	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\pi$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\pi$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\pi$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\pi$		
—	$-\frac{\sqrt{6}}{3}H_\pi^{11}$	$-\frac{\sqrt{6}}{3}H_\pi^{11}$	$-\frac{\sqrt{3}}{3}H_\pi^{11}$	$\frac{2\sqrt{3}}{3}H_\pi^{11}$	$-\frac{\sqrt{6}}{6}H_\pi^{11}$	$-\frac{\sqrt{30}}{6}H_\pi^{11}$		
—	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\rho$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\rho$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\rho$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\rho$		
$\chi_{b0}(n^3P_0)$	$-\frac{\sqrt{2}}{3}H_\rho^{11}$	$-\frac{\sqrt{2}}{3}H_\rho^{11}$	$-\frac{1}{3}H_\rho^{11}$	0	$-\frac{\sqrt{2}}{2}H_\rho^{11}$	$\frac{\sqrt{10}}{6}H_\rho^{11}$		
$\chi_{b1}(n^3P_1)$	$-\frac{\sqrt{6}}{6}H_\rho^{11}$	$-\frac{\sqrt{6}}{6}H_\rho^{11}$	$\frac{\sqrt{3}}{6}H_\rho^{11}$	0	$\frac{\sqrt{6}}{4}H_\rho^{11}$	$-\frac{\sqrt{30}}{12}H_\rho^{11}$		
$\chi_{b2}(n^3P_2)$	$-\frac{\sqrt{10}}{6}H_\rho^{11}$	$-\frac{\sqrt{10}}{6}H_\rho^{11}$	$\frac{\sqrt{5}}{6}H_\rho^{11}$	0	$\frac{\sqrt{10}}{4}H_\rho^{11}$	$-\frac{5\sqrt{2}}{12}H_\rho^{11}$		
Initial state		Final state $I^G(J^{PC}) = 1^-(1^{+-})$						
$\chi_{b1}(n^3P_1)$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)\pi$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)\pi$	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)\pi$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\pi$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\pi$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\pi$		
—	$\frac{2\sqrt{3}}{3}H_\pi^{11}$	$\frac{2\sqrt{3}}{3}H_\pi^{11}$	$-\frac{\sqrt{6}}{6}H_\pi^{11}$	0	$-\frac{\sqrt{3}}{2}H_\pi^{11}$	$\frac{\sqrt{15}}{6}H_\pi^{11}$		
—	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)\rho$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)\rho$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\rho$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\rho$		
$h_b(n^1P_1)$	$\frac{\sqrt{3}}{3}H_\rho^{11}$	$-\frac{\sqrt{3}}{3}H_\rho^{11}$	$\frac{\sqrt{6}}{3}H_\rho^{11}$	$\frac{\sqrt{6}}{3}H_\rho^{11}$	$\frac{\sqrt{3}}{3}H_\rho^{11}$	$\frac{\sqrt{15}}{3}H_\rho^{11}$		
Initial state		Final state $I^G(J^{PC}) = 1^+(1^{+-})$			Initial state		Final state $I^G(J^{PC}) = 1^-(1^{++})$	
$\Upsilon(n^3S_1)$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\pi$	$B^*\bar{B}^*\pi$			$\Upsilon(n^3S_1)$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\rho$		
$\Upsilon(n^3D_1)$	$-H_\pi^{00}$	H_π^{00}			$\Upsilon(n^3D_1)$	$-\frac{\sqrt{6}}{3}H_\rho^{10}$		
—	0	0			$\Upsilon(n^3D_2)$	$\frac{\sqrt{30}}{6}H_\rho^{12}$		
—						$-\frac{\sqrt{10}}{2}H_\rho^{12}$		
—	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\rho$	$B^*\bar{B}^*\rho$			—			
$\eta_b(n^1S_0)$	H_ρ^{10}	H_ρ^{10}						
$\eta_{b2}(n^1D_2)$	H_ρ^{12}	H_ρ^{12}						
Initial state		Final state $I^G(J^{PC}) = 0^-(1^{--})$						
$\Upsilon(n^3S_1)$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\sigma$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\sigma$	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\sigma$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\sigma$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\sigma$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\sigma$		
$\Upsilon(n^3D_1)$	0	0	0	0	0	0		
—	0	0	0	0	0	0		
—	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\eta$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\eta$	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\eta$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\eta$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\eta$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\eta$		
$h_b(n^1P_1)$	$\frac{\sqrt{6}}{3}H_\eta^{11}$	$-\frac{\sqrt{6}}{3}H_\eta^{11}$	$-\frac{\sqrt{3}}{3}H_\eta^{11}$	$\frac{2\sqrt{3}}{3}H_\eta^{11}$	$-\frac{\sqrt{6}}{6}H_\eta^{11}$	$-\frac{\sqrt{30}}{6}H_\eta^{11}$		
—	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\omega$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\omega$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\omega$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\omega$		
$\chi_{b0}(n^3P_0)$	$-\frac{\sqrt{2}}{3}H_\omega^{11}$	$-\frac{\sqrt{2}}{3}H_\omega^{11}$	$-\frac{1}{3}H_\omega^{11}$	0	$-\frac{\sqrt{2}}{2}H_\omega^{11}$	$\frac{\sqrt{10}}{6}H_\omega^{11}$		
$\chi_{b1}(n^3P_1)$	$-\frac{\sqrt{6}}{6}H_\omega^{11}$	$-\frac{\sqrt{6}}{6}H_\omega^{11}$	$\frac{\sqrt{3}}{6}H_\omega^{11}$	0	$\frac{\sqrt{6}}{4}H_\omega^{11}$	$-\frac{\sqrt{30}}{12}H_\omega^{11}$		
$\chi_{b2}(n^3P_2)$	$-\frac{\sqrt{10}}{6}H_\omega^{11}$	$-\frac{\sqrt{10}}{6}H_\omega^{11}$	$\frac{\sqrt{5}}{6}H_\omega^{11}$	0	$\frac{\sqrt{10}}{4}H_\omega^{11}$	$-\frac{5\sqrt{2}}{12}H_\omega^{11}$		
Initial state		Final state $I^G(J^{PC}) = 0^+(1^{+-})$						
$\chi_{b1}(n^3P_1)$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)\eta$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)\eta$	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)\eta$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\eta$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\eta$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\eta$		
—	$\frac{2\sqrt{3}}{3}H_\eta^{11}$	$\frac{2\sqrt{3}}{3}H_\eta^{11}$	$-\frac{\sqrt{6}}{6}H_\eta^{11}$	0	$-\frac{\sqrt{3}}{2}H_\eta^{11}$	$\frac{\sqrt{15}}{6}H_\eta^{11}$		
—	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)\omega$	$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)\omega$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\omega$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\omega$		
$h_b(n^1P_1)$	$\frac{\sqrt{3}}{3}H_\omega^{11}$	$-\frac{\sqrt{3}}{3}H_\omega^{11}$	$\frac{\sqrt{6}}{3}H_\omega^{11}$	$\frac{\sqrt{6}}{3}H_\omega^{11}$	$\frac{\sqrt{3}}{3}H_\omega^{11}$	$\frac{\sqrt{15}}{3}H_\omega^{11}$		
Initial state		Final state $I^G(J^{PC}) = 1^+(1^{+-})$			Initial state		Final state $I^G(J^{PC}) = 1^-(1^{++})$	
$h_b(n^1P_1)$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\sigma$	$B^*\bar{B}^*\sigma$			$\chi_{b1}(n^3P_1)$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\sigma$		
—	H_σ^{11}	H_σ^{11}			—	$-\frac{\sqrt{6}}{3}H_\sigma^{10}$		
—	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\eta$	$B^*\bar{B}^*\eta$						
$\Upsilon(n^3S_1)$	$-H_\eta^{00}$	H_η^{00}						
$\Upsilon(n^3D_1)$	0	0						
—								
—	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\omega$	$B^*\bar{B}^*\omega$				$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\omega$		
$\eta_b(n^1S_0)$	H_ρ^{10}	H_ρ^{10}			$\Upsilon(n^3S_1)$	$-\frac{\sqrt{6}}{3}H_\omega^{10}$		
$\eta_{b2}(n^1D_2)$	H_ρ^{12}	H_ρ^{12}			$\Upsilon(n^3D_1)$	$\frac{\sqrt{30}}{6}H_\omega^{12}$		
					$\Upsilon(n^3D_2)$	$-\frac{\sqrt{10}}{2}H_\omega^{12}$		

TABLE VIII: The typical relations between the decay widths $\Gamma((b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson})$ and the reduced matrix elements $H_{\alpha}^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$, where the indices i and j denote the light spin of the final and initial hadron respectively, and Q denotes the angular momentum of the final light meson.

Initial state		Final state $I^G(J^{PC}) = 1^+(2^{--})$			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\rho$	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)\rho$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\rho$	
$\chi_{b1}(n^3P_1)$	$-\frac{\sqrt{3}}{3}H_{\rho}^{11}$	$\frac{\sqrt{6}}{12}H_{\rho}^{11}$	$-\frac{1}{2}H_{\rho}^{11}$	$-\frac{\sqrt{6}}{4}H_{\rho}^{11}$	
$\chi_{b2}(n^3P_2)$	H_{ρ}^{11}	$-\frac{\sqrt{2}}{4}H_{\rho}^{11}$	$\frac{\sqrt{3}}{2}H_{\rho}^{11}$	$\frac{3\sqrt{2}}{4}H_{\rho}^{11}$	
Initial state		Final state $I^G(J^{PC}) = 1^-(2^{+-})$			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\pi$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\pi$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)\pi$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\pi$	
$\chi_{b2}(n^3P_2)$	$-\frac{2\sqrt{6}}{3}H_{\pi}^{11}$	$-\frac{\sqrt{3}}{6}H_{\pi}^{11}$	$-\frac{\sqrt{2}}{2}H_{\pi}^{11}$	$-\frac{\sqrt{3}}{2}H_{\pi}^{11}$	
$h_b(n^1P_1)$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\rho$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)\rho$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\rho$	
	0	0	0	0	
Initial state		Final state $I^G(J^{PC}) = 1^-(0^{+-})$			
	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)\pi$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\pi$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\pi$		
$\chi_{b1}(n^3P_1)$	$\sqrt{2}H_{\pi}^{11}$	$-\frac{\sqrt{6}}{3}H_{\pi}^{11}$	$\frac{2\sqrt{3}}{3}H_{\pi}^{11}$		
$h_b(n^1P_1)$	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)\rho$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\rho$		
	0	0	0		
Initial state		Final state $I^G(J^{PC}) = 1^+(0^{--})$			
	$\frac{1}{\sqrt{2}}(B_0\bar{B} - B\bar{B}_0)\rho$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\rho$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\rho$		
$\chi_{b1}(n^3P_1)$	H_{ρ}^{11}	$-\frac{\sqrt{3}}{3}H_{\rho}^{11}$	$-\frac{2\sqrt{6}}{3}H_{\rho}^{11}$		
Initial state		Final state $I^G(J^{PC}) = 1^-(2^{++})$		Initial state	
	$B^*\bar{B}^*\pi$			$B\bar{B}\pi$	$B^*\bar{B}^*\pi$
$\eta_{b2}(n^1D_2)$	0			$\frac{\sqrt{2}}{2}H_{\pi}^{00}$	$\frac{\sqrt{3}}{2}H_{\pi}^{00}$
	$B^*\bar{B}^*\rho$			$B\bar{B}\rho$	$B^*\bar{B}^*\rho$
$\Upsilon(n^3S_1)$	$-\frac{\sqrt{3}}{6}H_{\rho}^{10}$			$-\frac{\sqrt{2}}{2}H_{\rho}^{10}$	$\frac{\sqrt{6}}{6}H_{\rho}^{10}$
$\Upsilon(n^3D_1)$	$-\frac{\sqrt{5}}{10}H_{\rho}^{12}$			$-\frac{\sqrt{10}}{20}H_{\rho}^{12}$	$\frac{\sqrt{30}}{60}H_{\rho}^{12}$
$\Upsilon(n^3D_2)$	$\frac{\sqrt{50}}{10}H_{\rho}^{12}$				
$\Upsilon(n^3D_3)$	$-\frac{\sqrt{70}}{5}H_{\rho}^{12}$				
Initial state		Final state $I^G(J^{PC}) = 0^-(2^{--})$			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\sigma$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\sigma$	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)\sigma$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\sigma$	
$\Upsilon(n^3D_2)$	0	0	0	0	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\omega$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)\omega$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\omega$	
$\chi_{b1}(n^3P_1)$	$-\frac{\sqrt{3}}{3}H_{\omega}^{11}$	$\frac{\sqrt{6}}{12}H_{\omega}^{11}$	$-\frac{1}{2}H_{\omega}^{11}$	$-\frac{\sqrt{6}}{4}H_{\omega}^{11}$	
$\chi_{b2}(n^3P_2)$	H_{ω}^{11}	$-\frac{\sqrt{2}}{4}H_{\omega}^{11}$	$\frac{\sqrt{3}}{2}H_{\omega}^{11}$	$\frac{3\sqrt{2}}{4}H_{\omega}^{11}$	
Initial state		Final state $I^G(J^{PC}) = 0^+(2^{+-})$			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\sigma$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\sigma$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)\sigma$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\sigma$	
$\eta_{b2}(n^1D_2)$	0	0	0	0	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\eta$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\eta$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)\eta$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\eta$	
$\chi_{b2}(n^3P_2)$	$-\frac{2\sqrt{6}}{3}H_{\eta}^{11}$	$-\frac{\sqrt{3}}{6}H_{\eta}^{11}$	$-\frac{\sqrt{2}}{2}H_{\eta}^{11}$	$-\frac{\sqrt{3}}{2}H_{\eta}^{11}$	
$h_b(n^1P_1)$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\omega$	$\frac{1}{\sqrt{2}}(B_2\bar{B} + B\bar{B}_2)\omega$	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\omega$	
	0	0	0	0	
Initial state		Final state $I^G(J^{PC}) = 0^+(0^{+-})$			
	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)\sigma$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\sigma$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\sigma$		
$\eta_b(n^1S_0)$	0	0	0		
	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)\eta$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\eta$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\eta$		
$\chi_{b1}(n^3P_1)$	$\sqrt{2}H_{\eta}^{11}$	$-\frac{\sqrt{6}}{3}H_{\eta}^{11}$	$\frac{2\sqrt{3}}{3}H_{\eta}^{11}$		
$h_b(n^1P_1)$	$\frac{1}{\sqrt{2}}(B_0\bar{B} + B\bar{B}_0)\omega$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\omega$		
	0	0	0		
Initial state		Final state $I^G(J^{PC}) = 0^-(0^{--})$			
	$\frac{1}{\sqrt{2}}(B_0\bar{B} - B\bar{B}_0)\omega$	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\omega$	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\omega$		
$\chi_{b1}(n^3P_1)$	H_{ω}^{11}	$-\frac{\sqrt{3}}{3}H_{\omega}^{11}$	$-\frac{2\sqrt{6}}{3}H_{\omega}^{11}$		
Initial state		Final state $I^G(J^{PC}) = 0^+(2^{++})$		Initial state	
	$B^*\bar{B}^*\sigma$			$B\bar{B}\sigma$	$B^*\bar{B}^*\sigma$
$\chi_{b2}(n^3P_2)$	$-\frac{\sqrt{3}}{6}H_{\sigma}^{10}$			$\frac{\sqrt{6}}{2}H_{\sigma}^{11}$	$-\frac{\sqrt{2}}{2}H_{\sigma}^{11}$
	$B^*\bar{B}^*\eta$			$B\bar{B}\eta$	$B^*\bar{B}^*\eta$
$\eta_{b2}(n^1D_2)$	0			$\frac{\sqrt{2}}{2}H_{\eta}^{00}$	$\frac{\sqrt{3}}{2}H_{\eta}^{00}$
	$B^*\bar{B}^*\omega$			$B\bar{B}\omega$	$B^*\bar{B}^*\omega$
$\Upsilon(n^3S_1)$	$-\frac{\sqrt{3}}{6}H_{\omega}^{10}$			$-\frac{\sqrt{2}}{2}H_{\omega}^{10}$	$\frac{\sqrt{6}}{6}H_{\omega}^{10}$
$\Upsilon(n^3D_1)$	$-\frac{\sqrt{5}}{10}H_{\omega}^{12}$			$-\frac{\sqrt{10}}{20}H_{\omega}^{12}$	$\frac{\sqrt{30}}{60}H_{\omega}^{12}$
$\Upsilon(n^3D_2)$	$\frac{\sqrt{50}}{10}H_{\omega}^{12}$				
$\Upsilon(n^3D_3)$	$-\frac{\sqrt{70}}{5}H_{\omega}^{12}$				

TABLE IX: The typical ratios of the $(b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}$ decay widths.

Final state	$I^G(J^{PC})$	Initial state	
		$\Gamma(\chi_{b0}(n^3P_0)) : \Gamma(\chi_{b1}(n^3P_1)) : \Gamma(\chi_{b2}(n^3P_2))$	
$1^+(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\rho$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\rho$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\rho$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\rho$	0 : 0 : 0	
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\rho$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\rho$	4 : 3 : 5	
$1^-(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\rho$	$\Gamma(\Upsilon(n^3D_1)) : \Gamma(\Upsilon(n^3D_2))$ 1 : 3	
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\rho$	$\Gamma(\chi_{b1}(n^3P_1)) : \Gamma(\chi_{b2}(n^3P_2))$ 1 : 3	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\rho$	1 : 3	
	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)\rho$	1 : 3	
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\rho$	1 : 3	
	$1^-(2^{++})$ $B^*\bar{B}^*\rho$	$\Gamma(\Upsilon(n^3D_1)) : \Gamma(\Upsilon(n^3D_2)) : \Gamma(\Upsilon(n^3D_3))$ 1 : 15 : 84	
$0^-(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)\omega$	$\Gamma(\chi_{b0}(n^3P_0)) : \Gamma(\chi_{b1}(n^3P_1)) : \Gamma(\chi_{b2}(n^3P_2))$ 4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)\omega$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)\omega$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)\omega$	0 : 0 : 0	
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)\omega$	4 : 3 : 5	
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)\omega$	4 : 3 : 5	
$0^+(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\omega$	$\Gamma(\Upsilon(n^3D_1)) : \Gamma(\Upsilon(n^3D_2))$ 1 : 3	
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)\omega$	$\Gamma(\chi_{b1}(n^3P_1)) : \Gamma(\chi_{b2}(n^3P_2))$ 1 : 3	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)\omega$	1 : 3	
	$\frac{1}{\sqrt{2}}(B_2\bar{B} - B\bar{B}_2)\omega$	1 : 3	
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)\omega$	1 : 3	
	$0^+(2^{++})$ $B^*\bar{B}^*\omega$	$\Gamma(\Upsilon(n^3D_1)) : \Gamma(\Upsilon(n^3D_2)) : \Gamma(\Upsilon(n^3D_3))$ 1 : 15 : 84	

molecular/resonant states. Here, we only investigate the decay behaviors of those vector and axial vector states with $J^{PC} = 1^{--}, 1^{+-}, 1^{++}$ and 1^{++} . The typical ratios of the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B}^* \text{ or } B^*\bar{B}^*) \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B}^* \text{ or } B^*\bar{B}^*) + \text{light meson}$ decay widths depend on the following parameters

$$A' = \frac{H_{10}(m)}{H_{11}(m)}, \quad C' = \frac{H_{01}(m)}{H_{11}(m)}, \quad (23)$$

where $H_{10}(m) = \langle Q, 1 || H_{eff}(m) || 0 \rangle$, $H_{11}(m) = \langle Q, 1 || H_{eff}(m) || 1 \rangle$, and $H_{01}(m) = \langle Q, 0 || H_{eff}(m) || 1 \rangle$. m can be $\pi, \eta, \rho, \omega, \sigma$, while Q is the spin of the light meson.

If the reduced matrix elements satisfy the following relation

$$H_{01}(m) = -\frac{1}{\sqrt{3}}H_{10}(m),$$

the typical ratios satisfy the crossing symmetry, which is an important test of our calculation. We list the

obtained typical ratios of the $B_{(1,2)}\bar{B}^{(*)}(B\bar{B}^* \text{ or } B^*\bar{B}^*) \rightarrow B_{(1,2)}\bar{B}^{(*)}(B\bar{B}^* \text{ or } B^*\bar{B}^*) + \text{light meson}$ decay widths in Tables XI-XII.

IV. THE STRONG DECAYS OF THE HIDDEN-CHARM SYSTEMS

The above discussion of the strong decays or production behaviors of hidden beauty systems can be extended to investigate the strong decays of the hidden charm systems. In the following, we combine the experimental information of these observed charmonium-like states with our numerical results.

A. $Y(4260)$ and $Y(4360)$

The charmonium-like state $Y(4260)$ with $J^{PC} = 1^{--}$ was reported by the BaBar Collaboration in the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ process [33]. Assuming $Y(4260)$ to be the isoscalar state of

TABLE X: The typical ratios $\frac{\Gamma((b\bar{b}) \rightarrow B_{(1,2)} \bar{B}^{(*)} + \text{light meson})}{\Gamma((b\bar{b}) \rightarrow B_{(1,2)} \bar{B}^{(*)} + \text{light meson})}$, where the initial molecular states are different while the final states are the same.

Final state	$I^G(J^{PC})$	Initial state						
		$h_b(n^1P_1)$		$\chi_{b0}(n^3P_0)$	$\chi_{b1}(n^3P_1)$	$\chi_{b2}(n^3P_2)$		
$1^+(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\pi$	1 : 2	$1^+(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\rho$	4 : 0	4 : 0		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\pi$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\rho$				
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\pi$	1 : 5		$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\rho$	9 : 5	9 : 5		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\pi$			$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\rho$				
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\pi$	1 : 2		$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\rho$	4 : 0	4 : 0		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\pi$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\rho$	4 : 0	4 : 0		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\pi$	2 : 1		$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\rho$	2 : 9	2 : 9		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\pi$			$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\rho$				
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\pi$	1 : 1		$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\rho$	1 : 1	1 : 1		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\pi$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\rho$				
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\pi$	2 : 5		$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\rho$	2 : 5	2 : 5		
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\pi$			$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\rho$				
$0^-(1^{--})$		$\Upsilon(n^3S_1)$	$\Upsilon(n^3D_1)$		$\chi_{b0}(n^3P_0)$	$\chi_{b1}(n^3P_1)$	$\chi_{b2}(n^3P_2)$	
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\sigma$	0 : 0	0 : 0	$0^-(1^{--})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\omega$	4 : 0	4 : 0	4 : 0
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\sigma$				$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\omega$			
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\sigma$	0 : 0	0 : 0		$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\omega$	9 : 5	9 : 5	9 : 5
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\sigma$				$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\omega$			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\sigma$	0 : 0	0 : 0		$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\omega$	4 : 0	4 : 0	4 : 0
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\sigma$				$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\omega$			
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\sigma$	0 : 0	0 : 0		$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\omega$	2 : 9	2 : 9	2 : 9
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\sigma$				$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\omega$			
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\sigma$	0 : 0	0 : 0		$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\omega$	1 : 1	1 : 1	1 : 1
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\sigma$				$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\omega$			
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\sigma$	0 : 0	0 : 0		$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\omega$	2 : 5	2 : 5	2 : 5
$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\sigma$			$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\omega$					
$0^-(1^{--})$		$h_b(n^1P_1)$		$\chi_{b1}(n^3P_1)$				
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\eta$	1 : 2	$0^+(1^{+-})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\eta$	16 : 0			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\eta$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\eta$				
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\eta$	1 : 5		$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\eta$	9 : 5			
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\eta$			$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\eta$				
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\eta$	1 : 2		$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\eta$	16 : 0			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*+B^*\bar{B}'_1)\eta$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\eta$				
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\eta$	2 : 1		$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\eta$	2 : 9			
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*+B^*\bar{B}_1)\eta$			$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\eta$				
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*-B^*\bar{B}_0)\eta$	1 : 1		$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\eta$	1 : 1			
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}-B\bar{B}'_1)\eta$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\eta$				
	$\frac{1}{\sqrt{2}}(B_1\bar{B}-B\bar{B}_1)\eta$	2 : 5		$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\eta$	2 : 5			
$\frac{1}{\sqrt{2}}(B_2\bar{B}^*-B^*\bar{B}_2)\eta$		$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\eta$						
$1^-(1^{+-})$		$\chi_{b1}(n^3P_1)$		$h_b(n^1P_1)$		$h_b(n^1P_1)$		
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\pi$	16 : 0	$1^-(1^{+-})$	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\rho$	1 : 2	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\omega$	1 : 2	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\pi$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\rho$		$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\omega$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\pi$	9 : 5		$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\rho$	1 : 5	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\omega$	1 : 5	
	$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\pi$			$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\rho$		$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\omega$		
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\pi$	16 : 0		$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\rho$	1 : 2	$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\omega$	1 : 2	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\pi$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\rho$		$\frac{1}{\sqrt{2}}(B'_1\bar{B}^*-B^*\bar{B}'_1)\omega$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\pi$	2 : 9		$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\rho$	2 : 1	$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\omega$	2 : 1	
	$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\pi$			$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\rho$		$\frac{1}{\sqrt{2}}(B_1\bar{B}^*-B^*\bar{B}_1)\omega$		
	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\pi$	1 : 1		$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\rho$	1 : 1	$\frac{1}{\sqrt{2}}(B_0\bar{B}^*+B^*\bar{B}_0)\omega$	1 : 1	
	$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\pi$			$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\rho$		$\frac{1}{\sqrt{2}}(B'_1\bar{B}+B\bar{B}'_1)\omega$		
	$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\pi$	2 : 5		$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\rho$	2 : 5	$\frac{1}{\sqrt{2}}(B_1\bar{B}+B\bar{B}_1)\omega$	2 : 5	
$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\pi$		$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\rho$			$\frac{1}{\sqrt{2}}(B_2\bar{B}^*+B^*\bar{B}_2)\omega$			
$1^+(1^{+-})$		$\Upsilon(n^3S_1)$	$\Upsilon(n^3D_1)$		$\eta_b(n^1S_0)$	$\eta_{b2}(n^1D_2)$		
	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})\pi}{B^*\bar{B}^*\pi}$	1 : 1	0 : 0	$1^+(1^{+-})$	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})\rho}{B^*\bar{B}^*\rho}$	1 : 1	1 : 1	
	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})\sigma}{B^*\bar{B}^*\sigma}$	1 : 1		$0^-(1^{+-})$	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})\omega}{B^*\bar{B}^*\omega}$	1 : 1	1 : 1	
$0^-(1^{+-})$		$h_b(n^1P_1)$			$\eta_b(n^1S_0)$	$\eta_{b2}(n^1D_2)$		
	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})\sigma}{B^*\bar{B}^*\sigma}$	1 : 1						
$0^-(1^{+-})$		$\Upsilon(n^3S_1)$	$\Upsilon(n^3D_1)$					
	$\frac{1}{\sqrt{2}}\frac{(B\bar{B}^*-B^*\bar{B})\eta}{B^*\bar{B}^*\eta}$	1 : 1	0 : 0					

TABLE XI: The typical ratios $\frac{\Gamma(B_{(1,2)}\bar{B}^{(*)}\rightarrow B_{(1,2)}\bar{B}^{(*)}+light\ meson)}{\Gamma(B_{(1,2)}\bar{B}^{(*)}\rightarrow B_{(1,2)}\bar{B}^{(*)}+light\ meson)}$, where the initial molecular states are different while the final states are the same. The parameter A' is defined as $A' = \frac{H_{10}(m)}{H_{11}(m)}$.

Initial state $1^+(1^{--})$	$0^-(1^{+-}) + \pi$		$1^+(1^{+-}) + \eta$		$0^+(1^{++}) + \rho$	$1^-(1^{++}) + \omega$
	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\pi$	$B^*\bar{B}^*\pi$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\eta$	$B^*\bar{B}^*\eta$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\rho$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\omega$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	1 : 2	1 : 2	1 : 2	1 : 2	4 : 0	4 : 0
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	1 : 5	1 : 5	1 : 5	1 : 5	9 : 5	9 : 5
$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	1 : 2	1 : 2	1 : 2	1 : 2	4 : 0	4 : 0
$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	2 : 1	2 : 1	2 : 1	2 : 1	2 : 9	2 : 9
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	1 : 1	1 : 1	1 : 1	1 : 1	1 : 1	1 : 1
$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	2 : 5	2 : 5	2 : 5	2 : 5	2 : 5	2 : 5
Initial state $0^-(1^{--})$	$1^+(1^{+-}) + \pi$		$0^-(1^{+-}) + \eta$		$1^-(1^{++}) + \rho$	$0^+(1^{++}) + \omega$
	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\pi$	$B^*\bar{B}^*\pi$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\eta$	$B^*\bar{B}^*\eta$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\rho$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\omega$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	1 : 2	1 : 2	1 : 2	1 : 2	4 : 0	4 : 0
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	1 : 5	1 : 5	1 : 5	1 : 5	9 : 5	9 : 5
$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	1 : 2	1 : 2	1 : 2	1 : 2	4 : 0	4 : 0
$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	2 : 1	2 : 1	2 : 1	2 : 1	2 : 9	2 : 9
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	1 : 1	1 : 1	1 : 1	1 : 1	1 : 1	1 : 1
$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	2 : 5	2 : 5	2 : 5	2 : 5	2 : 5	2 : 5
Initial state $1^-(1^{+-})$	$0^+(1^{++}) + \pi$	$1^-(1^{++}) + \eta$	$0^-(1^{+-}) + \rho$		$1^+(1^{+-}) + \omega$	
	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\pi$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\eta$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\rho$	$B^*\bar{B}^*\rho$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\omega$	$B^*\bar{B}^*\omega$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	16 : 0	16 : 0	$\frac{(\sqrt{3}+2A')^2}{6}$	$\frac{(\sqrt{3}-2A')^2}{6}$	$\frac{(\sqrt{3}+2A')^2}{6}$	$\frac{(\sqrt{3}-2A')^2}{6}$
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	9 : 5	9 : 5	$\frac{(2\sqrt{3}-3A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{3}+3A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$	$\frac{(2\sqrt{3}-3A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{3}+3A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$
$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	16 : 0	16 : 0	$\frac{(\sqrt{3}-2A')^2}{6}$	$\frac{(\sqrt{3}+2A')^2}{6}$	$\frac{(\sqrt{3}-2A')^2}{6}$	$\frac{(\sqrt{3}+2A')^2}{6}$
$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	2 : 9	2 : 9	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{3}+3A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{3}-3A')^2}$	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{3}+3A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{3}-3A')^2}$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	1 : 1	1 : 1	$\frac{(\sqrt{3}+2A')^2}{(\sqrt{3}-2A')^2}$	$\frac{(\sqrt{3}-2A')^2}{(\sqrt{3}+2A')^2}$	$\frac{(\sqrt{3}+2A')^2}{(\sqrt{3}-2A')^2}$	$\frac{(\sqrt{3}-2A')^2}{(\sqrt{3}+2A')^2}$
$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	2 : 5	2 : 5	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$
Initial state $0^+(1^{+-})$	$1^-(1^{++}) + \pi$	$0^+(1^{++}) + \eta$	$1^+(1^{+-}) + \rho$		$0^-(1^{+-}) + \omega$	
	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\pi$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})\eta$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\rho$	$B^*\bar{B}^*\rho$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\omega$	$B^*\bar{B}^*\omega$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	16 : 0	16 : 0	$\frac{(\sqrt{3}+2A')^2}{6}$	$\frac{(\sqrt{3}-2A')^2}{6}$	$\frac{(\sqrt{3}+2A')^2}{6}$	$\frac{(\sqrt{3}-2A')^2}{6}$
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	9 : 5	9 : 5	$\frac{(2\sqrt{3}-3A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{3}+3A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$	$\frac{(2\sqrt{3}-3A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{3}+3A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$
$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	16 : 0	16 : 0	$\frac{(\sqrt{3}-2A')^2}{6}$	$\frac{(\sqrt{3}+2A')^2}{6}$	$\frac{(\sqrt{3}-2A')^2}{6}$	$\frac{(\sqrt{3}+2A')^2}{6}$
$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	2 : 9	2 : 9	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{3}+3A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{3}-3A')^2}$	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{3}+3A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{3}-3A')^2}$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$						
$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	1 : 1	1 : 1	$\frac{(\sqrt{3}+2A')^2}{(\sqrt{3}-2A')^2}$	$\frac{(\sqrt{3}-2A')^2}{(\sqrt{3}+2A')^2}$	$\frac{(\sqrt{3}+2A')^2}{(\sqrt{3}-2A')^2}$	$\frac{(\sqrt{3}-2A')^2}{(\sqrt{3}+2A')^2}$
$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$						
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	2 : 5	2 : 5	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$	$\frac{(2\sqrt{6}-\sqrt{2}A')^2}{(2\sqrt{15}+\sqrt{5}A')^2}$	$\frac{(2\sqrt{6}+\sqrt{2}A')^2}{(2\sqrt{15}-\sqrt{5}A')^2}$

TABLE XII: The typical ratios of the $B_{(1,2)}\bar{B}^{(*)} \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}$ decay widths. The parameter C' is defined as $C' = \frac{H_{01}(m)}{H_{11}(m)}$.

Initial state $1^-(1^{--})$	$0^-(1^{+-}) + \rho$	$1^+(1^{+-}) + \omega$
	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\rho : B^*\bar{B}^*\rho$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\omega : B^*\bar{B}^*\omega$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$	$\frac{(1-2C')^2}{(1+2C')^2}$	$\frac{(1-2C')^2}{(1+2C')^2}$
$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	$\frac{(1-2C')^2}{(1+2C')^2}$	$\frac{(1-2C')^2}{(1+2C')^2}$
$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$	$\frac{(2+C')^2}{(2-C')^2}$	$\frac{(2+C')^2}{(2-C')^2}$
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	$\frac{1}{1}$	$\frac{1}{1}$
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$\frac{(2+3C')^2}{(2-3C')^2}$	$\frac{(2+3C')^2}{(2-3C')^2}$
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	$\frac{(2-C')^2}{(2+C')^2}$	$\frac{(2-C')^2}{(2+C')^2}$
Initial state $0^+(1^{--})$	$1^+(1^{+-}) + \rho$	$0^-(1^{+-}) + \omega$
	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\rho : B^*\bar{B}^*\rho$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\omega : B^*\bar{B}^*\omega$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* + B^*\bar{B}_0)$	$\frac{(1-2C')^2}{(1+2C')^2}$	$\frac{(1-2C')^2}{(1+2C')^2}$
$\frac{1}{\sqrt{2}}(B'_1\bar{B} + B\bar{B}'_1)$	$\frac{(1-2C')^2}{(1+2C')^2}$	$\frac{(1-2C')^2}{(1+2C')^2}$
$\frac{1}{\sqrt{2}}(B_1\bar{B} + B\bar{B}_1)$	$\frac{(2+C')^2}{(2-C')^2}$	$\frac{(2+C')^2}{(2-C')^2}$
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* - B^*\bar{B}'_1)$	$\frac{1}{1}$	$\frac{1}{1}$
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* - B^*\bar{B}_1)$	$\frac{(2+3C')^2}{(2-3C')^2}$	$\frac{(2+3C')^2}{(2-3C')^2}$
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* + B^*\bar{B}_2)$	$\frac{(2-C')^2}{(2+C')^2}$	$\frac{(2-C')^2}{(2+C')^2}$
Initial state $1^+(1^{--})$	$0^-(1^{+-}) + \pi$	$1^+(1^{+-}) + \eta$
	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\pi : B^*\bar{B}^*\pi$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\eta : B^*\bar{B}^*\eta$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	$1 : 1$	$1 : 1$
Initial state $0^-(1^{--})$	$1^+(1^{+-}) + \pi$	$0^-(1^{+-}) + \eta$
	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\pi : B^*\bar{B}^*\pi$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})\eta : B^*\bar{B}^*\eta$
$\frac{1}{\sqrt{2}}(B_0\bar{B}^* - B^*\bar{B}_0)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B'_1\bar{B} - B\bar{B}'_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B_1\bar{B} - B\bar{B}_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B'_1\bar{B}^* + B^*\bar{B}'_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B_1\bar{B}^* + B^*\bar{B}_1)$	$1 : 1$	$1 : 1$
$\frac{1}{\sqrt{2}}(B_2\bar{B}^* - B^*\bar{B}_2)$	$1 : 1$	$1 : 1$

$\frac{1}{\sqrt{2}}(D_1\bar{D} - D\bar{D}_1)$ system [2, 43], we can write down the spin isospin wave function of $Y(4260)$ in heavy quark limit, i.e.,

$$\begin{aligned}
 |Y(4260)\rangle = & \frac{1}{\sqrt{2}} \left[\frac{\sqrt{6}}{6} (0_H^{-+} \otimes 1_l^{++})|_{J=1}^{-+} - \frac{\sqrt{3}}{6} (0_H^{-+} \otimes 1_l^{+-})|_{J=1}^{-+} \right. \\
 & + \frac{\sqrt{3}}{6} (1_H^{--} \otimes 1_l^{++})|_{J=1}^{--} - \frac{\sqrt{6}}{12} (1_H^{--} \otimes 1_l^{+-})|_{J=1}^{--} \\
 & + \frac{\sqrt{10}}{4} (1_H^{--} \otimes 1_l^{+-})|_{J=1}^{--} \left. \right] \left(\frac{|(c\bar{d})(\bar{c}d)\rangle + |(c\bar{u})(\bar{c}u)\rangle}{\sqrt{2}} \right) \\
 & + \frac{1}{\sqrt{2}} \left[\frac{\sqrt{6}}{6} (0_H^{-+} \otimes 1_l^{++})|_{J=1}^{-+} + \frac{\sqrt{3}}{6} (0_H^{-+} \otimes 1_l^{+-})|_{J=1}^{-+} \right. \\
 & - \frac{\sqrt{3}}{6} (1_H^{--} \otimes 1_l^{++})|_{J=1}^{--} - \frac{\sqrt{6}}{12} (1_H^{--} \otimes 1_l^{+-})|_{J=1}^{--} \\
 & + \frac{\sqrt{10}}{4} (1_H^{--} \otimes 1_l^{+-})|_{J=1}^{--} \left. \right] \left(\frac{|(\bar{c}d)(c\bar{d})\rangle + |(\bar{c}u)(c\bar{u})\rangle}{\sqrt{2}} \right).
 \end{aligned}$$

The discovery mode of $Y(4260)$ is $J/\psi\pi^+\pi^-$. If assuming the $\pi^+\pi^-$ pair in the final state is from the intermediate σ resonance, we write down the spin isospin wave function of $J/\psi\sigma$

$$|J/\psi\sigma\rangle = |(1_H^{--} \otimes 0_l^{++})_0^{--}\rangle |(c\bar{c})\rangle \frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u}),$$

In the heavy quark symmetry limit, we find this decay mode is suppressed.

However, the decay mode $\chi_{cJ}\omega$ is allowed with the spin structures of the final states

$$\begin{aligned} |\chi_{c0}\omega\rangle &= \left[\frac{1}{3}(1_H^{--} \otimes 0_l^{++})_{J=1}^{--} - \frac{\sqrt{3}}{3}(1_H^{--} \otimes 1_l^{++})_{J=1}^{--} \right. \\ &\quad \left. + \frac{\sqrt{5}}{3}(1_H^{--} \otimes 2_l^{++})_{J=1}^{--} \right] |(c\bar{c})\rangle \frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u}), \\ |\chi_{c1}\omega\rangle &= \left[-\frac{\sqrt{3}}{3}(1_H^{--} \otimes 0_l^{++})_{J=1}^{--} + \frac{1}{2}(1_H^{--} \otimes 1_l^{++})_{J=1}^{--} \right. \\ &\quad \left. + \frac{\sqrt{15}}{6}(1_H^{--} \otimes 2_l^{++})_{J=1}^{--} \right] |(c\bar{c})\rangle \frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u}), \\ |\chi_{c2}\omega\rangle &= \left[\frac{\sqrt{5}}{3}(1_H^{--} \otimes 0_l^{++})_{J=1}^{--} - \frac{\sqrt{15}}{6}(1_H^{--} \otimes 1_l^{++})_{J=1}^{--} \right. \\ &\quad \left. + \frac{1}{6}(1_H^{--} \otimes 2_l^{++})_{J=1}^{--} \right] |(c\bar{c})\rangle \frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u}). \end{aligned}$$

In the heavy quark symmetry, we obtain the ratio of the strong decays $Y(4260) \rightarrow \chi_{cJ}\omega$ ($J = 0, 1, 2$), i.e.,

$$\begin{aligned} \Gamma(\chi_{c0}\omega) : \Gamma(\chi_{c1}\omega) : \Gamma(\chi_{c2}\omega) \\ = 4 : 3 : 5, \end{aligned}$$

where the phase space factors are ignored. Since the ω meson can decay into $\pi^+\pi^-\pi^0$, then we can get the ratio of the strong decays $Y(4260) \rightarrow \chi_{cJ}\pi^+\pi^-\pi^0$ ($J = 0, 1, 2$),

$$\begin{aligned} \Gamma(\chi_{c0}\pi^+\pi^-\pi^0) : \Gamma(\chi_{c1}\pi^+\pi^-\pi^0) : \Gamma(\chi_{c2}\pi^+\pi^-\pi^0) \\ = 4 : 3 : 5 (2.11 : 1 : 1.28), \end{aligned}$$

where the ratio in the bracket is the result considering the phase space factors. This ratio can also be used to test whether $Y(4260)$ has the $\frac{1}{\sqrt{2}}(D_1\bar{D}^* - D\bar{D}_1)$ structure.

The Belle Collaboration reported that there exists a charmonium-like state $Y(4360)$ in the $\psi(2S)\pi^+\pi^-$ invariant mass spectrum of the $e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$ process [55]. $Y(4360)$ was suggested as an isoscalar state $\frac{1}{\sqrt{2}}(D_1\bar{D}^* + D^*\bar{D}_1)$

state [2]. Then, its spin isospin wave function is

$$\begin{aligned} |Y(4360)\rangle &= \frac{1}{\sqrt{2}} \left[\frac{\sqrt{3}}{6}(0_H^{--} \otimes 1_l^{++})_{J=1}^{--} - \frac{\sqrt{6}}{12}(0_H^{--} \otimes 1_l^{+-})_{J=1}^{--} \right. \\ &\quad + \frac{\sqrt{6}}{4}(1_H^{--} \otimes 1_l^{++})_{J=1}^{--} - \frac{\sqrt{3}}{4}(1_H^{--} \otimes 1_l^{+-})_{J=1}^{--} \\ &\quad \left. - \frac{\sqrt{5}}{4}(1_H^{--} \otimes 1_l^{+-})_{J=1}^{--} \right] \left(\frac{|(c\bar{d})(\bar{c}d)\rangle + |(c\bar{u})(\bar{c}u)\rangle}{\sqrt{2}} \right) \\ &\quad + \frac{1}{\sqrt{2}} \left[-\frac{\sqrt{3}}{6}(0_H^{--} \otimes 1_l^{++})_{J=1}^{--} - \frac{\sqrt{6}}{12}(0_H^{--} \otimes 1_l^{+-})_{J=1}^{--} \right. \\ &\quad + \frac{\sqrt{6}}{4}(1_H^{--} \otimes 1_l^{++})_{J=1}^{--} + \frac{\sqrt{3}}{4}(1_H^{--} \otimes 1_l^{+-})_{J=1}^{--} \\ &\quad \left. + \frac{\sqrt{5}}{4}(1_H^{--} \otimes 1_l^{+-})_{J=1}^{--} \right] \left(\frac{|(\bar{c}d)(cd)\rangle + |(\bar{c}u)(cu)\rangle}{\sqrt{2}} \right). \end{aligned}$$

The $\chi_{cJ}\omega$ are the allowed decay modes of $Y(4360)$. We have the following ratio

$$\begin{aligned} \Gamma(\chi_{c0}\omega) : \Gamma(\chi_{c1}\omega) : \Gamma(\chi_{c2}\omega) \\ = 4 : 3 : 5. \end{aligned}$$

We can get the ratio of these strong decays $Y(4360) \rightarrow \chi_{cJ}\pi^+\pi^-\pi^0$ ($J = 0, 1, 2$)

$$\begin{aligned} \Gamma(\chi_{c0}\pi^+\pi^-\pi^0) : \Gamma(\chi_{c1}\pi^+\pi^-\pi^0) : \Gamma(\chi_{c2}\pi^+\pi^-\pi^0) \\ = 4 : 3 : 5 (1.94 : 1 : 1.36). \end{aligned}$$

where we assume the 3π in the final states comes from the intermediate ω contribution. The results in the bracket include the phase space factors. We also suggest that future experiments carry out the measurement of this ratio, which can be applied to test the $\frac{1}{\sqrt{2}}(D_1\bar{D}^* + D^*\bar{D}_1)$ assignment of $Y(4360)$.

In addition, $h_c\eta$ and $\chi_{cJ}\omega$ are the allowed decay modes of both $Y(4260)$ and $Y(4360)$. The spin isospin wave function of $h_c\eta$ is

$$|h_c\eta\rangle = |(0_H^{--} \otimes 1_l^{+-})_0^{--}\rangle |(c\bar{c})\rangle \frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u}).$$

In the heavy quark limit, the D and D^* mesons belong to the same heavy spin multiplet. Hence, $Y(4260)$ and $Y(4360)$ have the same spatial wave functions and the same spatial matrix elements of these discussed strong decays, which leads to quite simple ratios between their decay widths, i.e.,

$$\begin{aligned} \frac{\Gamma(Y(4260) \rightarrow h_c\eta)}{\Gamma(Y(4360) \rightarrow h_c\eta)} &= 2 : 1 (1.65 : 1), \\ \frac{\Gamma(Y(4260) \rightarrow \chi_{c0}\pi^+\pi^-\pi^0)}{\Gamma(Y(4360) \rightarrow \chi_{c0}\pi^+\pi^-\pi^0)} &= 2 : 9 (1 : 6.39), \\ \frac{\Gamma(Y(4260) \rightarrow \chi_{c1}\pi^+\pi^-\pi^0)}{\Gamma(Y(4360) \rightarrow \chi_{c1}\pi^+\pi^-\pi^0)} &= 2 : 9 (1 : 6.98), \\ \frac{\Gamma(Y(4260) \rightarrow \chi_{c2}\pi^+\pi^-\pi^0)}{\Gamma(Y(4360) \rightarrow \chi_{c2}\pi^+\pi^-\pi^0)} &= 2 : 9 (1 : 7.39), \end{aligned}$$

where the results in the brackets are from the consideration of the phase space factors.

B. $X(3872)$

There were extensive discussions of $X(3872)$ as an isoscalar $D\bar{D}^*$ molecular state with $J^{PC} = 1^{++}$ [22–31]. In this picture, its spin isospin wave function reads as

$$\begin{aligned} |X(3872)\rangle &= \frac{1}{\sqrt{2}} \left[\frac{1}{2} (0_H^{--} \otimes 1_l^{--})_{J=1}^{+-} - \frac{1}{2} (1_H^{--} \otimes 0_l^{--})_{J=1}^{+-} \right. \\ &\quad + \frac{1}{\sqrt{2}} (1_H^{--} \otimes 1_l^{--})_{J=1}^{++} \left. \left(\frac{|(c\bar{d})(\bar{c}d)\rangle + |(c\bar{u})(\bar{c}u)\rangle}{\sqrt{2}} \right) \right] \\ &\quad + \frac{1}{\sqrt{2}} \left[-\frac{1}{2} (0_H^{--} \otimes 1_l^{--})_{J=1}^{+-} + \frac{1}{2} (1_H^{--} \otimes 0_l^{--})_{J=1}^{+-} \right. \\ &\quad + \frac{1}{\sqrt{2}} (1_H^{--} \otimes 1_l^{--})_{J=1}^{++} \left. \left(\frac{|(\bar{c}d)(c\bar{d})\rangle + |(\bar{c}u)(c\bar{u})\rangle}{\sqrt{2}} \right) \right]. \end{aligned}$$

$\psi(1^3D_1)\omega$ and $\psi(1^3D_2)\omega$ are its kinematically forbidden modes. The $\chi_{c1}\sigma$ and $J/\psi\omega$ modes are allowed with the corresponding spin isospin wave functions

$$\begin{aligned} |\chi_{c1}\sigma\rangle &= |(1_H^{--} \otimes 1_l^{--})_1^{++}\rangle |(c\bar{c})\rangle \frac{1}{\sqrt{2}} (d\bar{d} + u\bar{u}), \\ |J/\psi\omega\rangle &= (1_H^{--} \otimes 1_l^{--})_{J=1}^{++} |(c\bar{c})\rangle \frac{1}{\sqrt{2}} (d\bar{d} + u\bar{u}). \end{aligned}$$

In the heavy quark symmetry limit, we find that the decay modes $\chi_{c1}\sigma$ and $J/\psi\omega$ are related to the spin configurations $(1_H^{--} \otimes 1_l^{--})_1^{++}$ and $(1_H^{--} \otimes 1_l^{--})_{J=1}^{++}$.

C. $Z_c(3900)$ and $Z_c(4020)$

$Z_c(3900)$ was first reported by BESIII in the $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ process at $\sqrt{s} = 4.26$ GeV [56, 57], which was suggested as the charged isovector state of the $D\bar{D}^*$ system with $I^G(J^P) = 1^+(1^+)$. $Z_c(4020)$ was observed in the $h_c\pi^\pm$ invariant mass spectrum of $e^+e^- \rightarrow h_c\pi^+\pi^-$ at $\sqrt{s} = 4.26$ GeV [58]. A similar state $Z_c(4025)$ was reported by BESIII in $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$ at $\sqrt{s} = 4.26$ GeV [59]. $Z_c(4020)$ (or $Z_c(4025)$) may be the charged isovector state of $D^*\bar{D}^*$ system with $I^G(J^P) = 1^+(1^+)$ [49]. The spin isospin wave functions

of their neutral partners read as

$$\begin{aligned} |Z_c(3900)\rangle &= \frac{1}{\sqrt{2}} \left[\frac{1}{2} (0_H^{--} \otimes 1_l^{--})_{J=1}^{+-} - \frac{1}{2} (1_H^{--} \otimes 0_l^{--})_{J=1}^{+-} \right. \\ &\quad + \frac{1}{\sqrt{2}} (1_H^{--} \otimes 1_l^{--})_{J=1}^{++} \left. \left(\frac{|(c\bar{d})(\bar{c}d)\rangle - |(c\bar{u})(\bar{c}u)\rangle}{\sqrt{2}} \right) \right] \\ &\quad - \frac{1}{\sqrt{2}} \left[-\frac{1}{2} (0_H^{--} \otimes 1_l^{--})_{J=1}^{+-} + \frac{1}{2} (1_H^{--} \otimes 0_l^{--})_{J=1}^{+-} \right. \\ &\quad + \frac{1}{\sqrt{2}} (1_H^{--} \otimes 1_l^{--})_{J=1}^{++} \left. \left(\frac{|(\bar{c}d)(c\bar{d})\rangle - |(\bar{c}u)(c\bar{u})\rangle}{\sqrt{2}} \right) \right], \\ |Z_c(4020)\rangle &= \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} (0_H^{--} \otimes 1_l^{--})_{J=1}^{+-} + \frac{1}{\sqrt{2}} (1_H^{--} \otimes 0_l^{--})_{J=1}^{+-} \right] \\ &\quad \times \left(\frac{|(c\bar{d})(\bar{c}d)\rangle - |(c\bar{u})(\bar{c}u)\rangle}{\sqrt{2}} \right) \\ &\quad + \left(\frac{|(\bar{c}d)(c\bar{d})\rangle - |(\bar{c}u)(c\bar{u})\rangle}{\sqrt{2}} \right). \end{aligned}$$

If ignoring the heavy quark symmetry, their allowed decay modes are $J/\psi\pi$, $\psi(1^3D_1)\pi$ and $\eta_c\rho$ with the spin structures

$$\begin{aligned} |J/\psi\pi^0\rangle &= |(1_H^{--} \otimes 0_l^{--})_0^{+-}\rangle |(c\bar{c})\rangle \frac{1}{\sqrt{2}} (d\bar{d} - u\bar{u}), \\ |\psi(1^3D_1)\pi^0\rangle &= |(1_H^{--} \otimes 2_l^{--})_0^{+-}\rangle |(c\bar{c})\rangle \frac{1}{\sqrt{2}} (d\bar{d} - u\bar{u}), \\ |\eta_c\rho^0\rangle &= (0_H^{--} \otimes 1_l^{--})_{J=1}^{+-} |(c\bar{c})\rangle \frac{1}{\sqrt{2}} (d\bar{d} - u\bar{u}). \end{aligned}$$

Considering the heavy quark symmetry, we find that the decay mode $\psi(1^3D_1)\pi$ is suppressed. While $J/\psi\pi$, $\eta_c\rho$ and $\eta_{c2}\rho$ are still allowed, which are consistent with the conclusion in Ref. [49]. We obtain the ratios between the decay widths of $Z_c(3900)$ and $Z_c(4020)$, i.e.,

$$\begin{aligned} \frac{\Gamma(Z_c(3900) \rightarrow J/\psi\pi^0)}{\Gamma(Z_c(4020) \rightarrow J/\psi\pi^0)} &= 1 : 1 (1 : 1.07), \\ \frac{\Gamma(Z_c(3900) \rightarrow \eta_c\rho^0)}{\Gamma(Z_c(4020) \rightarrow \eta_c\rho^0)} &= 1 : 1 (1 : 2.47), \end{aligned}$$

where the values in the brackets are the results after considering the phase space factors.

D. $Y(4274)$

$Y(4274)$ was observed in the $J/\psi\phi$ invariant mass spectrum by CDF [60], which can be as an S-wave isoscalar state $D_s\bar{D}_{s0}(2317)$ with $J^{PC} = 0^{-+}$ [61], whose spin structure reads

as

$$2. \quad J^{PC} = 2^{++}$$

$$\begin{aligned} |Y(4274)\rangle = & \left[-\frac{1}{2}(0_H^{++} \otimes 0_l^{--})|_{J=0}^{--} + \frac{1}{2}(1_H^{--} \otimes 1_l^{++})|_{J=0}^{--} \right. \\ & + \frac{\sqrt{2}}{2}(1_H^{--} \otimes 1_l^{+-})|_{J=0}^{+-} \Big] |c\bar{s}; \bar{c}s\rangle \\ & - \left[\frac{1}{2}(0_H^{++} \otimes 0_l^{--})|_{J=0}^{--} - \frac{1}{2}(1_H^{--} \otimes 1_l^{++})|_{J=0}^{--} \right. \\ & + \frac{\sqrt{2}}{2}(1_H^{--} \otimes 1_l^{+-})|_{J=0}^{+-} \Big] |\bar{c}s; c\bar{s}\rangle. \end{aligned}$$

In the heavy quark spin symmetry, $Y(4274)$ can decay into $J/\psi\phi$ via the P-wave transition, where the spin configuration $(1_H^{--} \otimes 1_l^{+-})|_{J=0}^{+-}$ is dominant with the decay width proportional to the reduced matrix element $|\langle 1, 0 || H_{eff}(\phi) || 1 \rangle|$.

E. $Y(3940)$ and $Y(4140)$

$Y(3940)$ was observed by the BaBar Collaboration [62] in $B \rightarrow KJ/\psi\omega$, while the CDF Collaboration observed $Y(4140)$ in $B \rightarrow KJ/\psi\phi$ [63]. $Y(3940)$ and $Y(4140)$ can be the candidates of the $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ molecular systems, respectively [64, 65]. Their quantum numbers may be $J^{PC} = 0^{++}$ or $J^{PC} = 2^{++}$ [64, 65]. In the following, we discuss their decay behaviors in these two cases.

1. $J^{PC} = 0^{++}$

If $Y(3940)$ and $Y(4140)$ are the 0^{++} molecular states, their spin structures read as

$$\begin{aligned} |Y(3940)\rangle &= \left[\frac{\sqrt{3}}{2}(0_H^{++} \otimes 0_l^{--})|_{J=0}^{++} - \frac{1}{2}(1_H^{--} \otimes 1_l^{--})|_{J=0}^{++} \right] \\ &\quad \times \left(\frac{|\langle c\bar{d} \rangle \langle \bar{c}d \rangle| - |\langle c\bar{u} \rangle \langle \bar{c}u \rangle|}{\sqrt{2}} + \frac{|\langle \bar{c}d \rangle \langle c\bar{d} \rangle| - |\langle \bar{c}u \rangle \langle c\bar{u} \rangle|}{\sqrt{2}} \right), \\ |Y(4140)\rangle &= \left[\frac{\sqrt{3}}{2}(0_H^{++} \otimes 0_l^{--})|_{J=0}^{++} - \frac{1}{2}(1_H^{--} \otimes 1_l^{--})|_{J=0}^{++} \right] |c\bar{s} \rangle \langle \bar{c}s|. \end{aligned}$$

The kinematically allowed decay modes of $Y(3940)$ are $\chi_{c0}\sigma$, $\eta_c\eta$ and $J/\psi\omega$, whose decay widths are proportional to the reduced matrix elements $|\langle 0, 1 || H_{eff}(\sigma) || 1 \rangle|$, $|\langle 0, 0 || H_{eff}(\eta) || 0 \rangle|$ and $|\langle 1, 2 || H_{eff}(\omega) || 1 \rangle|$, respectively. $Y(4140)$ can decay into $J/\psi\phi$ through the spin configuration $(1_H^{--} \otimes 1_l^{--})|_{J=0}^{++}$, whose decay width is proportional to the reduced matrix element $|\langle 1, 0 || H_{eff}(\phi) || 1 \rangle|$.

If both $Y(3940)$ and $Y(4140)$ are the 2^{++} tensor states, we can write down their spin structures in heavy quark limit, i.e.,

$$\begin{aligned} |Y(3940)\rangle &= (1_H^{--} \otimes 1_l^{--})|_{J=2}^{++} \\ &\quad \times \left(\frac{|\langle c\bar{d} \rangle \langle \bar{c}d \rangle| - |\langle c\bar{u} \rangle \langle \bar{c}u \rangle|}{\sqrt{2}} + \frac{|\langle \bar{c}d \rangle \langle c\bar{d} \rangle| - |\langle \bar{c}u \rangle \langle c\bar{u} \rangle|}{\sqrt{2}} \right), \\ |Y(4140)\rangle &= (1_H^{--} \otimes 1_l^{--})|_{J=2}^{++} |c\bar{s} \rangle \langle \bar{c}s|. \end{aligned}$$

The kinematically allowed decay modes of $Y(3940)$ are $\chi_{c2}\sigma$, $J/\psi\omega$ and $\eta_{c2}\eta$. In the heavy quark symmetry limit, the decay modes $\chi_{c2}\sigma$ and $J/\psi\omega$ are allowed with their widths proportional to the reduced matrix elements $|\langle 0, 0 || H_{eff}(\sigma) || 1 \rangle|$ and $|\langle 1, 0 || H_{eff}(\omega) || 1 \rangle|$ respectively. The decay mode $\eta_{c2}\eta$ is suppressed due to the conservations of heavy and light spin. In this case, $Y(4140)$ can also decay into $J/\psi\phi$ through the spin configuration $(1_H^{--} \otimes 1_l^{--})|_{J=2}^{++}$, whose decay width is proportional to the reduced matrix element $|\langle 1, 0 || H_{eff}(\phi) || 1 \rangle|$.

V. SUMMARY

More and more charmonium-like and bottomium-like states were reported in the past twelve years. Many of them are very close to the open-charm or open-bottom threshold. Some are even charged. Many the so-called XYZ states do not fit into the traditional quark model spectrum easily. Many theoretical speculations were proposed to understand their inner structures. Among them, the molecule picture is quite popular. Historically, the deuteron has been identified to be a very loosely bound molecular state composed of a proton and neutron. It is very natural to investigate whether the loosely bound di-meson molecular states exist or not. Dynamical calculation based on the one boson exchange model may explore the possible existence of the di-meson molecular states. On the other hand, the decay pattern and production mechanism of these di-meson systems may also shed light on their inner structures.

Under the heavy quark symmetry, the QED and QCD interactions don't flip the heavy quark spin. The conservation of the heavy spin together with the isospin, total angular momentum and other quantum numbers such as parity, C parity and G parity provide an effective scheme to probe the inner structures of the XYZ states through their decay and production behaviors. We have extensively discussed the three classes of strong decays $B_{(1,2)}\bar{B}^{(*)} \rightarrow (b\bar{b}) + \text{light meson}$, $(b\bar{b}) \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}$, $B_{(1,2)}\bar{B}^{(*)} \rightarrow B_{(1,2)}\bar{B}^{(*)} + \text{light meson}$, corresponding to the strong decays of one molecular (resonant) state into a bottomonia, one bottomonia into a molecular (resonant) state, and strong decays of one molecular (resonant) state into another respectively. With the same formalism, we also give detailed discussions on the possible hidden-charm molecules (resonances).

If either the initial systems or final states belong to the same heavy spin multiplet, the spatial matrix elements of these

strong decays are the same, which leads to quite simple ratios between their decay widths. Different assumptions of the underlying structures will give different decay ratios and different production behaviors, which will help probe the inner structures of the XYZ states after comparison with experiment measurements. For instance, there are theoretical speculations that $Y(4260)$ may be an isoscalar $\frac{1}{\sqrt{2}}(D_1\bar{D} - D\bar{D}_1)$ molecule. In the heavy quark symmetry limit, $Y(4260)$ does not decay into $J/\psi\pi^+\pi^-$. In other words, the discovery mode $J/\psi\pi^+\pi^-$ of $Y(4260)$ disfavors the $\frac{1}{\sqrt{2}}(D_1\bar{D} - D\bar{D}_1)$ molecule scheme.

In short summary, the strong decay behaviors of the XYZ states encode important information on their underlying structures. Systematical experimental measurement of these decay behaviors will be helpful to judge the various theoretical interpretations of the XYZ states. Hopefully the present extensive

investigations will be useful to illuminate the future strong decay data.

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